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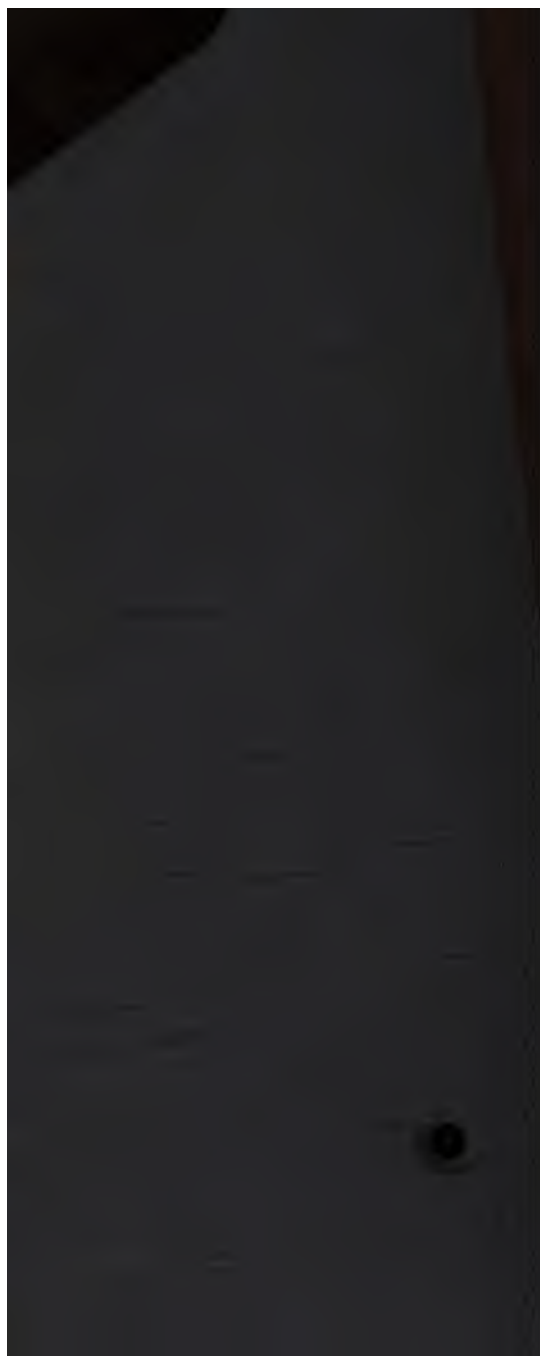
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FIRST STEPS IN NUMBERS

A PRIMARY ARITHMETIC

BY

WENTWORTH & REED

TEACHER'S EDITION.

PART III. (THIRD YEAR) NUMBERS ABOVE
TWENTY.

EducT 118.85.880(3)



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FIRST STEPS IN NUMBER.

BY
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TEACHER'S EDITION.
PART III.—THIRD YEAR: NUMBERS ABOVE TWENTY.

BOSTON:
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1885.

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PREFACE.

THE object of this book is to provide teachers with a record of the work done in number in the primary schools of to-day.

There has been no attempt at novelty in the subject-matter, in the arrangement of work, or in the manner of presentation. The whole is on a constructive basis. Numbers are chief; processes subordinate. What has been found to be more easily understood precedes the more difficult, without respect to its scientific relation. Fractions present no greater difficulty than wholes, so they accompany the teaching of integral numbers from the beginning. The law of dependence has been carefully observed, although at first glance the arrangement may not seem to warrant this assertion.

The object of every teacher is so to present numbers that the mind of the child may grasp firmly the facts concerning them, and hold these facts tenaciously by the law of association. Success lies in requiring the child to *show* what he is talking about, and in following the "step by step" rule. The book illustrates these two principles. It abounds in examples which have not before appeared in print, and which are calculated to interest the child from their close connection with his varied experiences. It gives suggestions for versatility of drill, and illustrates in detail the teaching of a hundred topics.

It is expected that the work to the number ten will be taken in one year, the work to twenty in another year, and the remainder of the course outlined in the book will be covered in two years more.

A child's book accompanies this edition, which the child may use with great advantage after he becomes acquainted with figures.

It is hoped that this book will find a welcome among all persons interested in leading children by easy and sure paths to a knowledge of numbers.

G. A. WENTWORTH.
E. M. REED.



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PRIMARY ARITHMETIC.

Teacher's Edition.

PART III.

CHAPTER XXII.

THE NUMBER TWENTY-ONE.

§ 44. THE NUMBER TWENTY-ONE.

I have six three-cent pieces; how much money have I?
If you give me another three-cent piece, how much money shall I have? How many three-cent pieces shall I have? Seven three-cent pieces are how much money then?

Seven oranges at three cents each will cost how much?

In seven yards of rope how many feet of rope?

Seven triangles have together how many sides?

Seven three-cornered hats with a tassel on each corner have together how many tassels?

If it takes three yards of ribbon for one pair of bonnet-strings, how many yards of ribbon will be required for seven pairs of bonnet-strings?

If Willie's hens lay three eggs a day, how many eggs will they lay in a week?

Express: Seven threes are twenty-one.

If you must put seven threes together to make twenty-one, how many threes can you find in twenty-one?

Twenty-one feet of chain are equal to how many yards of chain?

Twenty-one cents will buy how many three-cent pencils?

If I cut a strip of paper that is twenty-one inches long into pieces three inches long, into how many pieces shall I cut the strip of paper?

How many three-cent books can be bought for twenty-one cents?

If the news-boy sells seven papers at three cents apiece, how many cents does he receive for the papers? If he is allowed to keep one cent for each paper he sells, how many cents must he return to his employer? Fourteen and seven then are how many?

How many sevens make fourteen? and since fourteen and seven are twenty-one, how many sevens make twenty-one?

In three weeks how many days?

If there are three groups of stars and seven stars in each group, how many stars are in the three groups?

Three pounds of beef at seven cents a pound will cost how much?

Three pairs of horses at seven hundred dollars a pair will cost how many hundred dollars?

Express: Three sevens are twenty-one.

If three sevens make twenty-one, into how many sevens can twenty-one be divided? Twenty-one cents then will buy how many seven-cent loaves of bread? how many seven-cent copy-books?

If berries are seven cents a quart, how many quarts can be bought for twenty-one cents?

In twenty-one days from to-day I shall go home. How many Sundays before I go home? How many Mondays?

Twenty-one children received stars for singing nicely. Each seven received a different colored star. How many different stars were given out?

Express: Twenty-one divided by seven are three.

What three equal numbers make twenty-one?

What then is one-third of twenty-one?

If twenty-one nuts were divided equally among three boys, how many nuts did each boy receive?

A man who owed twenty-one dollars agreed to pay a third of the debt each month until it was paid. How much money did he pay each month?

If a bicycle will travel twenty-one miles in three hours, at what rate is that per hour?

Express: One-third of twenty-one is seven.

Count by threes to twenty-one.

Count by sevens to twenty-one.

Express: Eighteen and three; four and seven; fourteen and seven; eight and three; twenty-one minus three; eleven minus three; twenty-one minus seven; eleven minus seven.

Fill out:

$7 \times 3 =$	$21 - 7 =$	$\times 3 = 21$	$+ 3 = 21$
$21 \div 3 =$	$21 - 3 =$	$\times 7 = 21$	$+ 7 = 21$
$3 \times 7 =$	$14 + 7 =$	$+ 7 = 3$	$+ 10 = 21$
$21 \div 7 =$	$18 + 3 =$	$+ 3 = 7$	$+ 20 = 21$
$\frac{1}{3}$ of 21 =	$20 + 1 =$	$- 7 = 14$	$- 3 = 18$

Review Exercise.

Cloth that is a yard and a foot wide is how many feet wide?

If a sail cloth is seven yards long, how many feet long is it?

If a ship has twenty-one feet of rope in one line, how many yards long is the line?

How many days in a school week?

How many days in four school weeks?

If a dog has four toes on each foot, how many toes has he altogether?

If a squirrel has five toes on each fore-foot, how many toes have two squirrels on their fore-feet?

If a cat has five toes on each fore-foot, how many toes have two cats on their fore-feet?

A cat has four toes on each hind foot and five toes on each fore-foot. How many toes has she altogether?

If I cut ribbon which is eighteen inches long into pieces six inches long, how many such pieces shall I have?

In twenty-one days how many Sundays? how many Fridays? how many Saturdays?

How many gills in a quart? How many gills in two quarts?

If you go to school nine months in a year, how many months do you go to school in two years?

If one spider has nine eyes, how many eyes have two spiders?

If one parasol has ten ribs, how many ribs have two parasols? three parasols? seven parasols?

Name a number made up of ten and four; of ten and eight; of ten and two; of ten and seven; of four tens and seven; of six tens and six; of seven tens and nine; of nine tens and two; of eight tens and five; of five tens and eight; of three tens and nine; of two tens and one.

If there are three books on one shelf, and five times as many books on another shelf, how many books are on the second shelf?

If I slide down hill ten times, and you slide down hill twice as many times, how many times do you slide down hill?

If I have to walk a fourth of a mile to school, and you have to walk three times as far, what part of a mile do you walk to school?

If you have one-third of a dollar, and I have twice as much money, how many thirds of a dollar do I have?

If a family eats a quarter of a lamb each week for four weeks, how many quarters of lamb does the family eat?

If it takes a sixth of a yard of ribbon to make one loop, how much ribbon will it take for four loops?

If you have one-eighth of a dollar, and I have eight times as much money, how much money do I have?

How many two-cent cakes can you buy for five cents, and how many cents will you have left? for seven cents, and what will you have left? for nine cents? for seventeen cents? for nineteen cents? for twenty-one cents? for twenty-three cents?

How many three-cent pencils can you buy for four cents, and how many cents will you have left? for five cents, and how much will you have left? for eight cents? for eleven cents? for sixteen cents? for nineteen cents? for twenty-three cents?

How many fours and what else make nine? make thirteen? make eighteen? make twenty-one? make twenty-two?

Eleven divided by five gives what answer? Thirteen divided by five gives what answer? Sixteen divided by five? eighteen divided by five? twenty-one divided by five? twenty-three divided by five?

Divide eight by six. Divide by six: nine; ten; eleven; fourteen; sixteen; nineteen; twenty-one; twenty-three.

Copy and complete:

$7 = 3 \times 2 +$	$19 = 6 \times 3 +$	$7 = 1 \times 5 +$
$9 = 4 \times 2 +$	$23 = 7 \times 3 +$	$9 = 1 \times 5 +$
$13 = 6 \times 2 +$	$7 = 1 \times 4 +$	$13 = 2 \times 5 +$
$17 = 8 \times 2 +$	$11 = 2 \times 4 +$	$17 = 3 \times 5 +$
$21 = 10 \times 2 +$	$15 = 3 \times 4 +$	$21 = 4 \times 5 +$
$5 = 1 \times 3 +$	$17 = 4 \times 4 +$	$8 = 1 \times 6 +$
$7 = 2 \times 3 +$	$19 = 4 \times 4 +$	$13 = 2 \times 6 +$
$11 = 3 \times 3 +$	$21 = 5 \times 4 +$	$15 = 2 \times 6 +$

CHAPTER XXIII.

THE NUMBER TWENTY-FOUR.

§ 45. THE NUMBER TWENTY-FOUR.

If I have seven three-cent pieces, how much money do I have? If I earn another three-cent piece, how much money shall I have? Eight three-cent pieces, then, are how much money?

Eight three-cent postage stamps will cost how much? Eight three-cent pencils? Eight oranges at three cents apiece?

Eight clover leaves have how many leaflets?

Eight chestnut burrs contain how many nuts, if there are three nuts in each burr?

Eight yards of curtain-cord are how many feet of curtain-cord?

If a blackboard consists of eight slates, each a yard long, how many feet long is the blackboard?

It takes three yards of cloth to make a jacket. How many yards of cloth are required for eight jackets?

Express: Eight threes are twenty-four.

Who knows into how many threes we can divide twenty-four?

When lemons are three cents apiece, how many lemons can be bought for twenty-four cents?

A stick that is twenty-four feet long can be cut into how many yard-sticks?

A farmer's wife, who has twenty-four pounds of butter to make into balls, puts three pounds in each ball. How many balls of butter does she make?

A market-man measured out twenty-four quarts of berries with a three-quart measure. How many times did he fill the measure?

Express: Twenty-four divided by three are eight.

What eight equal numbers make twenty-four?

What part, then, of twenty-four are three?

If I divide twenty-four apples equally among eight boys, how many apples shall I give to each boy?

A man bought eight chairs for twenty-four dollars. What was the cost of each chair?

I put twenty-four pears away in the drawer to ripen, but an eighth of the number decayed. How many decayed?

This line is twenty-four inches long. How long is one-eighth of the line?

There are twenty-four hours in a day. How long is one-eighth of the day?

Jennie has twenty-four examples to do. When she has done three, what part of the number has she done?

Express: One-eighth of twenty-four is three.

Five fours are how many? Another four will make how many? How many fours make twenty-four, then?

Six squares have how many corners?

Six horses have how many legs?

Six dogs have how many legs?

Six gallons of oysters are how many quarts of oysters?

Six four-quart pails will hold how many quarts of syrup?

Six pints of yeast are equal to how many gills of yeast?

I bought six yards of silk at four dollars a yard. How much did I pay for the silk?

My little brother is four years old. I am six times as old as he. How old am I?

It is four miles to Newtown, and six times as far to Oldtown. How far is it to Oldtown?

Express: Six fours are twenty-four.

A chair-maker turned out twenty-four chair legs. How many chairs would they supply?

In twenty-four gills how many pints?

In twenty-four quarts how many gallons?

There were twenty-four buttons on a card, four in each row. How many rows of buttons were there?

Here are twenty-four splints of equal length. How many separate squares can you form from them?

Express: Twenty-four divided by four are six.

What six equal numbers make twenty-four?

What, then, is one-sixth of twenty-four?

If you have twenty-four buttons, and use a sixth of them for your boot, how many buttons do you use? How many buttons will you have left?

A man who owned twenty-four acres of land planted one-sixth of it with potatoes. How many acres of potatoes did he plant?

He planted another sixth of the land with corn. How many acres of corn had he?

If you earn twenty-four cents a week, what do you earn in a day.

The cook used twenty-four eggs in making six loaves of cake. How many eggs did she use for each loaf?

Express: One-sixth of twenty-four is four.

Give me a story for any fact in twenty-four which you have learned.

Tell me the answer as I point to each blank:—

$8 \times 3 =$	$6 \times 4 =$	$20 + = 24$	$24 - = 20$
$24 \div 3 =$	$24 \div 4 =$	$21 + = 24$	$24 - = 18$
$\frac{1}{2}$ of 24 =	$\frac{1}{3}$ of 24 =	$18 + = 24$	$24 - = 21$

Three sixes are how many? Another six will make how many? How many sixes make twenty-four?

How many dollars will a man earn in four weeks, if he earns a dollar a day?

If you go to school six days in a week, how many days will you go to school in four weeks?

How many links in a chain that is four feet long, if there are six links in each foot of the chain?

What will four car-fares cost at six cents a fare?

If six persons can sit on one settee, how many persons can sit on four settees?

Express: Four sixes are twenty-four.

Who has a story for twenty-four divided by six?

If eggs are packed in layers of half a dozen each, how many layers will twenty-four eggs make?

How many quarts of milk at six cents a quart can be bought for twenty-four cents?

How many window draperies can be made from twenty-four yards of goods, if six yards are required for one window?

Express: Twenty-four divided by six are four.

If twenty-four is divided into four equal groups, how many are in each group? What part of twenty-four is each group? What is one-fourth of twenty-four?

If four oranges cost twenty-four cents, what will one orange cost?

If four lamp-chimneys cost twenty-four cents, what will one lamp-chimney cost?

A boy who had twenty-four cents spent a fourth of his money for a balloon. How much money had he left?

There are twenty-four hours in a day. How many hours in a fourth of the day?

If the time between breakfast and dinner is a fourth of the day, how many hours is it between breakfast and dinner?

If I am in school a fourth of the day, how many hours am I in school?

If papa goes to work at seven o'clock, and comes home at one o'clock, what part of the day has he been at work?

There are twenty-four sheets of paper in a quire of paper. How many sheets in a quarter of a quire? If paper is twenty-four cents a quire, what will a sheet of paper cost? What will a fourth of a quire cost?

A quire of paper is usually put up in four equal lots. How many sheets of paper in each lot?

Express: One-fourth of twenty-four is six.

Write: Twenty-four sheets of paper make a quire of paper.

Which are more, eight threes or three eights? Eight threes are how many? Then three eights are how many?

Six fours are equal to how many eights? Six fours are how many? Then three eights are how many?

Who has an example for: Three eights are twenty-four?

Three oxen wear how many shoes?

Three hens have how many toes?

How many wheels have three passenger-cars?

Three pairs of horses need shoeing all round. How many shoes will be required to shoe them?

Express: Three eights are twenty-four.

If you should arrange twenty-four books on a book-case, putting eight books on each shelf, how many shelves would it take?

Twenty-four cents will buy how many loaves of bread at eight cents a loaf? How many yards of cambric at eight cents a yard?

Twenty-four quarts of molasses will fill how many two-gallon jugs?

Twenty-four gills of jelly will make how many quarts of jelly?

Express: Twenty-four divided by eight are three.

What three equal numbers make twenty-four?

What then is one-third of twenty-four?

A boy who had twenty-four brackets for sale, sold one-third of them. How many did he sell?

There were twenty-four spoons in the spoon-holder. We used a third of them at tea. How many spoons remained in the spoon-holder?

I had a quire of paper, but have used a third of it. How many sheets have I used?

Allen sleeps a third of the day, works a third of the day, and spends the rest of his time in amusement. How many hours has he for amusement?

Express: One-third of twenty-four is eight.

Take this number work:

$24 \div 8 =$	$3 \times 8 =$	$24 - 3 =$
$3 \times 8 =$	$8 \times 3 =$	$24 - 4 =$
$\frac{1}{3}$ of 24 =	$4 \times 6 =$	$24 - 6 =$
$24 \div 6 =$	$6 \times 4 =$	$24 - 8 =$
$4 \times 6 =$	$24 \div 3 =$	$8 \times \quad = 24$
$\frac{1}{4}$ of 24 =	$24 \div 8 =$	$6 \times \quad = 24$
$24 \div 4 =$	$24 \div 4 =$	$3 \times \quad = 24$
$6 \times 4 =$	$24 \div 6 =$	$4 \times \quad = 24$
$\frac{1}{5}$ of 24 =	$10 \div 4 =$	$24 \div \quad = 3$
$24 \div 3 =$	$20 \div 4 =$	$24 \div \quad = 4$
$8 \times 3 =$	$11 \div 3 =$	$24 \div \quad = 6$
$\frac{1}{6}$ of 24 =	$21 \div 3 =$	$24 \div \quad = 8$
$\frac{1}{8}$ of 24 =	$8 \div 6 =$	of 24 = 6
$\frac{1}{4}$ of 24 =	$18 \div 6 =$	of 24 = 4
$\frac{1}{3}$ of 24 =	$6 \div 8 =$	of 24 = 3
$\frac{1}{2}$ of 24 =	$16 \div 8 =$	of 24 = 8

24	24	24	14	24	14
— 10,	— 12	— 8	— 8	— 6	— 6
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
3	4	3	2	3	4
4	5	3	2	8	4
6	4	8	6	3	3
3	3	4	5	4	4
7	8	6	7	5	9
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

One-fifth.

Into how many parts have I cut the circle? (Five.) How do the parts compare in size? (Equal.) Who knows what part of the whole circle each one of these five equal parts is? Show me then one-fifth of the circle.

You may draw a line on the board, and divide it into fifths.

Divide this triangle into fifths by drawing lines from the vertex to the base.

What number can you divide into fifths and have *one* for each fifth? What number can you divide and have *two* for each fifth? What number has *three* in each fifth? What five equal numbers make twenty? Then what is one-fifth of twenty?

If I give you two two-cent pieces, and tell you that is one-fifth of the money I have, can you tell me how much money I have? Suppose I give you a three-cent piece, and tell you *that* is one-fifth of the money I have; can you tell me how much money I have?

Two is one-fifth of what number?

How many fifths make a whole?

CHAPTER XXIV.

THE NUMBER TWENTY-FIVE.

§ 46. THE NUMBER TWENTY-FIVE.

Twenty and five are how many? How many fives make twenty? Twenty and five then are how many fives?

Express: Five fives are twenty-five.

How many school-days in five weeks?

How many fingers have five gloves?

Five bureaus, with five drawers in each bureau, have how many drawers together?

How many feet of linen must I buy for five window-shades if each shade must be five feet long?

Each think of an example for: Five times five are twenty-five..

Each think of an example for: Twenty-five divided by five.

What is one of the five equal numbers which make twenty-five? What part of twenty-five is five?

If I had a quarter of a dollar, and you had one-fifth as much money, what piece of money would you have? If you had two-fifths as much money, what piece of money would you have? If you had three-fifths as much money, what two pieces of money would you have? If you had four five-cent pieces, what part of my quarter of a dollar would you have?

How many five-cent pieces must you have to equal my quarter?

Give an example for: One-fifth of twenty-five is five.

Express: One-fifth of twenty-five is five.

Tell me what number is needed in each blank:

$5 \times 5 =$	$\frac{1}{5}$ of 20 =	$\frac{1}{5} + \frac{1}{5} =$ 5.
$25 \div 5 =$	$\frac{1}{5}$ of 15 =	$\frac{2}{5} + \frac{1}{5} =$ 5.
$\frac{1}{5}$ of 25 =	$\frac{1}{5}$ of 10 =	$\frac{2}{5} + \frac{3}{5} =$?

Exercise for Review.

How many pints in a quart? How many pints in two quarts? four quarts? six quarts? eight quarts? ten quarts? eleven quarts? twelve quarts?

Eleven two-cent pieces are how much money? Eleven pairs of scissors have how many blades? Eleven sheets of paper have how many leaves? Eleven books have how many covers?

Express: Eleven twos are twenty-two.

Here are some blocks, each two inches thick. If eleven blocks are piled up, one top of the other, how high will the pile be?

This stick is two feet long. Eleven such sticks placed end to end would extend how many feet? How many yards? Twelve sticks placed end to end would extend how many feet? how many yards?

Twelve two-cent pieces are how much money? Twelve two-cent rolls of lozenges will cost how much money?

Express: Twelve twos are twenty-four.

Who has a story for twelve twos?

Express:

Once two.	Five twos.	Nine twos.
Two twos.	Six twos.	Ten twos.
Three twos.	Seven twos.	Eleven twos.
Four twos.	Eight twos.	Twelve twos.

CHAPTER XXV.

THE NUMBER TWENTY-SEVEN.

§ 47. THE NUMBER TWENTY-SEVEN.

Four and three are how many?

Fourteen and three are how many?

Twenty-four and three are how many?

Eight threes and what number make twenty-seven?

Then nine threes make what number?

Express: Nine threes are twenty-seven.

Nine three-cent pieces are how much money?

Nine three-cent pencils cost how much?

Nine triangles have how many sides?

Nine yards of chain are how many feet of chain?

Johnny is three years old. His papa is nine times as old. How old is his papa?

Here are twenty-seven short sticks. How many separate triangles can you form with them?

A chair-maker put three bars across the back of each chair. How many chairs would twenty-seven bars supply?

Twenty-seven feet of rope are how many yards of rope?

In each season there are three months. How many seasons are there in twenty-seven months?

A man who worked on a farm received three dollars a week. How many weeks did he work to earn twenty-seven dollars?

Express: Twenty-seven divided by three are nine.

Which is more, nine threes or three nines?

If, then, nine threes are twenty-seven, how much are three nines?

If there are nine school-months in a year, how many school-months in three years?

Three pounds of beef at nine cents a pound will cost how much?

Three yards of cambric at nine cents a yard will cost how much?

I can buy crackers at nine cents a dozen. How many dozen can I buy for twenty-seven cents?

How many quires of paper at nine cents a quire can I buy for twenty-seven cents?

What part of twenty-seven is nine?

If I divide twenty-seven plums equally among three children, how many plums shall I give to each?

If three pounds of dates cost twenty-seven cents, what does one pound cost?

If it takes a man twenty-seven days to lay a certain amount of pipe, how long will it take him to lay one-third as much pipe?

If it takes one man twenty-seven days to do a piece of work, how long will it take three men?

Read, and give the answers:

$9 \times 3 =$	$3 \times \quad = 27$	$\times 9 = 27$
$27 \div 3 =$	$9 \times \quad = 27$	$+ 9 = 3$
$3 \times 9 =$	$27 \div \quad = 3$	$\times 3 = 27$
$27 \div 9 =$	$27 \div \quad = 9$	$\div 3 = 9$
$\frac{1}{3}$ of 27 =	$18 + \quad = 27$	$+ 9 = 27$
$24 + 3 =$	$3 + \quad = 27$	$+ 24 = 27$
$\begin{array}{r} 3 \\ 9 \end{array}$	$\begin{array}{r} 9 \\ 3 \end{array}$	$\begin{array}{r} 18 \\ + 9 \\ \hline 27 \end{array}$
$\begin{array}{r} 27 \\ - 9 \\ \hline 18 \end{array}$	$\begin{array}{r} 27 \\ - 3 \\ \hline 24 \end{array}$	$\begin{array}{r} 27 \\ - 10 \\ \hline 17 \end{array}$
$\begin{array}{r} 27 \\ - 20 \\ \hline 7 \end{array}$		

Exercise for Review.

How many in a half-dozen? How many in two half-dozens? Which is more, two half-dozens or one dozen?

How many months in half a year? How many months in two halves of a year? Which is the longer time, two halves of a year or a whole year?

How many sheets of paper in half a quire? In two halves of a quire? Which is more, two halves or a whole?

How many halves in a whole?

How many inches in a third of a foot? How many inches in three-thirds of a foot?

How many eggs in a third of a dozen? In three-thirds of a dozen? Which is more, three-thirds of a dozen or a whole dozen?

How many hours in a third of a day? In three-thirds of a day? Which is the longer time, three-thirds of a day or a whole day?

How many thirds in a whole?

How many sheets of paper in a fourth of a quire? In four-fourths of a quire? Which is more, four-fourths of a quire or a whole quire?

How many months in a fourth of a year? In four-fourths of a year? Which is longer, four-fourths of a year or a whole year?

How many fourths in a whole?

Express: One-half of twelve is *six*.

One-third of twelve is *four*.

One-fourth of twelve is *three*.

One-sixth of twelve is *two*.

Which is most, one-half, one-third, one-fourth, or one-sixth of twelve? Which is least, one-half, one-third, one-fourth, or one-sixth of twelve? Which is more, one-third or one-fourth of twelve?

Express : One-half of twenty-four is twelve.

One-third of twenty-four is eight.

One-fourth of twenty-four is six.

One-sixth of twenty-four is four.

One-eighth of twenty-four is three.

Which is most, one-half, one-third, one-fourth, one-sixth, or one-eighth of twenty-four? Which is least, one-half, one-third, one-fourth, one-sixth, or one-eighth of twenty-four? Which is more, one-third or one-sixth of twenty-four?

Which is more, one-half of sixteen or one-eighth of sixteen? One-half of eighteen or one-sixth of eighteen? One-third of eighteen or one-sixth of eighteen?

Which is more, one-half of anything or one-third of the same thing? One-half or one-fourth? One-half or one-sixth? One-half or one-eighth? One-third or one-fourth? One-third or one-sixth? One-sixth or one-eighth?

Here is a large cube cut up into eight small cubes. How do the small cubes compare in size? (Equal.) Show me one-half of the cube. Show me one-fourth of the cube. How many fourths of the cube can you find in one-half the cube?

How many fourths of an apple does it take to equal one-half of an apple? How many fourths of an orange to equal half an orange?

One-half of a dollar equals how many fourths of a dollar? One-half of a day equals how many fourths of a day?

Write : One-half equals two-fourths. ($\frac{1}{2} = \frac{2}{4}$.)

Read and give the answers : —

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad \frac{3}{4} - \frac{1}{4} = \frac{2}{4} \quad 1 - \frac{1}{2} = \frac{2}{4} \quad \frac{2}{8} + \frac{1}{8} = ?$$

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4} \quad \frac{3}{4} + \frac{1}{4} = ? \quad 1 - \frac{3}{8} = \frac{5}{8} \quad \frac{3}{4} + \frac{1}{4} = ?$$

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \quad 1 - \frac{1}{4} = \frac{3}{4} \quad 1 - \frac{4}{8} = \frac{4}{8} \quad \frac{5}{8} + \frac{1}{8} = ?$$

$$\frac{3}{4} - \frac{1}{2} = \frac{1}{4} \quad 1 - \frac{3}{4} = \frac{1}{4} \quad 1 - \frac{2}{8} = \frac{6}{8} \quad \frac{7}{8} + \frac{1}{8} = ?$$

Look at this divided cube again. Into how many small cubes is it divided? (Eight.) How do the small cubes compare in size? (Equal.) Show me half of the cube. Show me one-eighth of the cube. How many eighths of the cube can you find in a half of the cube?

Here is a circle. Tell me into what it is divided. (Eighths.) Show me one-half of the circle. Show me one-eighth of the circle. How many eighths of the circle can you find in one-half of the circle?

Here is a square marked off on the board. Into what is it divided by the lines that I have drawn through it? (Eighths.) Show me half of the square. Show me one-eighth of the square. How many eighths of the square can you find in half of the square?

One-half of a pie equals how many eighths of the pie?

One half of a dollar equals how many eighths of a dollar?

Write: One-half equals four-eighths.

Copy and complete:

$$\begin{array}{llll} \frac{2}{8} + \frac{2}{8} = \frac{4}{8} & \frac{1}{2} + \frac{2}{8} = \frac{6}{8} & \frac{1}{2} + \frac{3}{8} = \frac{7}{8} & \frac{1}{2} - \frac{1}{8} = \frac{4}{8} \\ \frac{1}{2} - \frac{2}{8} = \frac{4}{8} & \frac{1}{2} + \frac{1}{8} = \frac{5}{8} & \frac{1}{2} + \frac{4}{8} = ? & \frac{1}{2} - \frac{3}{8} = \frac{1}{8} \end{array}$$

You may take the divided cube and find how many eighths of the cube there are in a fourth of the cube.

Here is an apple that is divided. Find how many eighths of the apple it takes to make a fourth of the apple.

Here is a circle that is divided. How many eighths of the circle equal a fourth of the circle?

How many eighths of an orange must you have to equal my fourth of an orange?

How many eighths of a mile equal one-fourth of a mile?

Write: One-fourth equals two-eighths.

Copy and complete:—

$$\begin{array}{llll} \frac{1}{8} + \frac{1}{8} = \frac{2}{8} & \frac{1}{2} = \frac{4}{8} & \frac{1}{4} = \frac{2}{8} & \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \\ \frac{1}{4} - \frac{1}{8} = \frac{1}{8} & \frac{1}{2} = \frac{8}{8} & \frac{3}{8} = \frac{3}{8} & \frac{1}{2} + \frac{1}{8} = \frac{5}{8} \end{array}$$

CHAPTER XXVI.

THE NUMBER TWENTY-EIGHT.

§ 48. THE NUMBER TWENTY-EIGHT.

Four and four are how many?

Fourteen and four are how many?

Twenty-four and four are how many?

Six fours and four are how many?

Seven fours then are how many?

Seven chairs have how many legs?

Seven wagons have how many wheels?

If a cask holds seven gallons, how many quarts does it hold?

If a pail holds seven pints, how many gills does it hold?

A sheet of note paper has four pages. Seven sheets of note paper have how many pages?

If there are four panes of glass in one window, how many panes of glass are there in seven windows?

Seven yokes of oxen have how many horns?

Twenty-eight horse-shoes will shoe how many horses all round?

Twenty-eight horses will supply how many four-horse barges?

How many oranges at four cents apiece can be bought for twenty-eight cents?

In twenty-eight weeks how many moons, if there is a moon every four weeks?

One and seven are how many?

Eleven and seven are how many?

Twenty-one and seven are how many?

Three sevens and seven are how many?

Four sevens are how many?

In four weeks how many days?

Four quarts of peanuts at seven cents a quart will cost how much?

Give me an example for: Four sevens are twenty-eight.

There are twenty-eight days in the month of February. How many weeks are there in February?

If a horse travels seven miles an hour, how many hours will it take him to travel twenty-eight miles?

Twenty-eight dollars will buy how many pairs of boots at seven dollars a pair?

Two dozen and a third of a dozen eggs are packed in layers of seven each. How many layers are there?

A little mill-sweeper earns seven cents every half-day. How many days must he work to earn twenty-eight cents?

What is one-fourth of twenty-eight?

If I have a quarter of a dollar and a three-cent piece, and you have a fourth as much money as I, how much money have you?

There were twenty-eight days in February. Three-fourths of the days were stormy. How many days were stormy?

If there are twenty-eight slats in a blind, how many slats are there in three-fourths of the blind?

In a long ladder there were twenty-eight rounds, but a fourth of the rounds got broken. How many rounds were there then in the ladder?

If I cut a stick of wood which is two feet and four inches long into sticks each seven inches long, how many sticks shall I make? How many cuts shall I make?

What numbers shall I need for these blanks?

$7 \times 4 =$	$21 + 7 =$	$28 \div = 4$
$28 \div 4 =$	$14 + 14 =$	$28 \div = 7$
$4 \times 7 =$	$24 + 4 =$	$4 \times = 28$
$28 \div 7 =$	$20 + 8 =$	$7 \times = 28$
$\frac{1}{4}$ of 28 =	$18 + 10 =$	$14 + = 28$
$\begin{array}{r} 28 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ - 21 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ - 4 \\ \hline \end{array}$
$\begin{array}{r} 28 \\ - 24 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ - 10 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ - 14 \\ \hline \end{array}$

Exercise for Review.

Count by twos to twenty-eight.

Count by threes to twenty-seven.

Count by fours to twenty-eight.

Count by fives to twenty-five.

Count by sixes to twenty-four.

Count by sevens to twenty-eight.

Count by eights to twenty-four.

Count by nines to twenty-seven.

Count by tens to one hundred.

How many dozen and what part of a dozen bananas are there in a bunch of twenty-eight bananas?

How many score and how many more chickens have I, if I have twenty-eight chickens?

How many quarts and what part of a quart of jelly in twenty-eight gills?

How many gallons of oil in a tank containing twenty-eight quarts?

How many years and what part of a year in twenty-eight months?

How many days and what part of a day in twenty-eight hours?

How many yards and how many feet besides in twenty-eight feet of rope?

Twenty-eight cents will buy how many two-cent postage stamps? How many three-cent pencils and what will remain? How many yards of curtain-cord at four cents a yard? How many dozen peaches at five cents a dozen, and what will remain? How many quires of paper at six cents a quire, and what will remain? How many cakes of maple sugar at seven cents a cake? How many quarts of blueberries at eight cents a quart, and what will remain? How many pounds of crackers at nine cents a pound, and what will remain? How many loaves of bread at ten cents a loaf, and what will remain? How many dozen pens at eleven cents a dozen, and what will remain?

If a boy works every second day in a week and earns a dollar a day, how many weeks must he work to earn twenty-seven dollars?

If I pay ten cents for a bottle of ink which holds a gill, and fifty cents for a quart bottle of ink, how much do I save in buying ink by the quart?

If you sleep one-third of the day and I sleep one-fourth of the day, who sleeps the more hours? How many more hours?

If John eats a half-dozen peaches and Mary eats a quarter of a dozen, who eats the more peaches? How many more?

If I write a third of the day to-day, and a sixth of the day to-morrow, how many more hours shall I write to-day than to-morrow?

Which is the larger part, a half of an apple or a third of the same apple? A half of a stick of candy or a third of the same stick of candy?

Into how many parts will you divide anything to divide it into halves? to divide it into thirds? to divide it into fourths? to divide it into fifths? into sixths? into eighths?

CHAPTER XXVII.

THE NUMBER THIRTY.

§ 49. THE NUMBER THIRTY.

Twenty-five and five are how many?

Five fives and five are how many?

Six fives are how many?

Six bunches of envelopes at five cents a bunch will cost how much?

If you have five cents and I have six times as much, how much money do I have?

Annie is five years old and her mamma is six times as old. How old is her mamma?

Thirty cents will buy how many bananas at five cents apiece?

In thirty school-days how many weeks of school?

A garden is thirty feet long and five feet broad; how many times its breadth is its length?

Nellie's little reading-book has thirty pages. If she reads five pages a week, in how many weeks will she finish the reader?

What is one of the six equal numbers into which thirty can be divided? What number then is one-sixth of thirty?

What piece of money is one-sixth of thirty cents?

If six bottles of ink cost thirty cents, what does one bottle cost?

If six spools of thread cost thirty cents, what does one spool cost?

James found five eggs early in the morning; but that was only one-sixth of the number he found before night. How many did he find in all?

Express: Six fives are thirty.

Thirty divided by five are six.

One-sixth of thirty is five.

Twenty-four and six are how many?

Twenty-four and six are how many sixes?

How many working days in five weeks?

How many balloons has a man who has five bunches with half a dozen in each bunch?

A ship that makes a voyage every six months will make how many voyages in thirty months?

A boy who earns a cent a day must work how many weeks to earn thirty cents?

A boy who earns six dollars a month must work how many months to earn thirty dollars?

What is one of the five equal numbers which make thirty?

What is one-fifth of thirty?

What two equal pieces of money make one-fifth of thirty?

One-fifth of thirty months is what part of a year?

One-fifth of thirty eggs is what part of a dozen eggs?

One-fourth of a quire of paper is what part of thirty sheets of paper?

One-half a foot is what part of thirty inches?

Read and give the answers:

$\frac{1}{3}$ of 30 =	$\times 5 = 30$	$30 \div = 10$
$\frac{1}{5}$ of 30 =	$\times 3 = 30$	$30 \div = 3$
$\frac{1}{6}$ of 30 =	$\times 6 = 30$	$30 \div = 6$
$30 \div 10 =$	$\times 10 = 30$	$30 \div = 5$
$30 \div 6 =$	of 30 = 10	$30 \div = 15$

$30 + 5 =$	$\text{of } 30 = 6$	$30 - = 20$
$3 \times 10 =$	$\text{of } 30 = 5$	$30 - = 15$
$5 \times 6 =$	$\text{of } 30 = 15$	$30 - = 24$
$6 \times 5 =$	$+ 6 = 30$	$30 - = 25$
30 30 30 30 30 30		
2 3 - 10 - 15 - 20 + 40		
<hr/>	<hr/>	<hr/>

Exercise for Review.

When anything is divided into five equal parts, what is each part called? What are four of the parts called?

When anything is divided into seven equal parts, what is each part of the whole? What part of the whole are three parts? four parts? seven parts?

What do you mean by one-eighth of anything? by three-eighths of anything? by two-fifths of anything? by five fifths of anything? by one-sixth of anything?

I have a half a dollar and Annie has a quarter of a dollar. Which has the more money? How much more money have I than Annie?

How many quarters of a dollar does it take to equal half a dollar?

I am in school a quarter of the day. Sadie is in school only one-eighth of the day. How many times as many hours am I in school as Sadie?

There are sixteen ounces in a pound. I have here two weights. One is one-fourth of a pound, the other is one-eighth of a pound. How many times as heavy is one as the other?

I have here a half-pound weight. How many eighth-pound weights will it take to balance it?

One-half of a pound of candy will make how many packages of candy each weighing an eighth of a pound?

How many blocks of chocolate, each weighing an eighth of a pound, must you put together to make a half a pound of chocolate? to make a quarter of a pound of chocolate?

In our climate snow is on the ground a third of the year, and only a sixth of the year is warm summer weather. How many times as much winter as summer have we?

Each loaf of ginger-bread takes a sixth of a dozen eggs. How many loaves of ginger-bread can be made from a third of a dozen eggs?

How many sixths of an apple does it take to equal a third of an apple?

A third of a yard will make how many sixths of a yard?

It snowed four hours to-day; what part of the day has it snowed? If it had snowed twice as many hours of the day, what part of the day would it have snowed?

Nellie ate a third of a pound of dates. If I ate half the quantity she ate, what part of a pound did I eat?

If a third of a pound of maple sugar costs six cents, what will a sixth of a pound cost? what will a half a pound cost?

Take this number-work:

$\frac{1}{2} = \frac{4}{8}$	$\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$	$\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$	$\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$
$\frac{1}{2} = \frac{4}{8}$	$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$	$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$	$\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$
$\frac{1}{2} = \frac{6}{12}$	$\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$	$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$	$\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$
$\frac{1}{4} = \frac{2}{8}$	$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$	$\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$	$\frac{2}{8} + \frac{2}{8} = \frac{4}{8}$
$\frac{1}{8} = \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$	$\frac{1}{8} - \frac{1}{8} = 0$	$\frac{2}{8} + \frac{2}{8} = \frac{4}{8}$

What will five pairs of boots cost at five dollars a pair?

On Mamie's fifth birthday she invited five children to visit her; each child invited was five years old. What was the sum of the little folks' ages, counting in Mamie?

In a caravan there were four rows of camels and five camels in each row. How many camels were in the caravan?

A boy received three five-cent pieces for shovelling the snow from a sidewalk. How much money did he receive?

Count by fives as far as you can.

How many fives make thirty? make fifteen? make twenty-five? make twenty?

Count by threes to thirty.

How many threes make twelve? make eighteen? make twenty-four? make fifteen? make thirty? make twenty-one?

Twenty gills of milk will cost how much at four cents a pint?

Twenty-four quarts of wine will cost how much at four dollars a gallon?

In sixteen weeks how many new moons, if there is a new moon every four weeks?

If I have twenty-eight quarters of dollars, how many dollars have I?

Five fours are how many? three fours? six fours? four fours? seven fours?

Count by fours to twenty-eight.

How many faces have two cubes?

How many sides have three boxes?

How many sides has a room, counting the floor and the ceiling? How many sides have four rooms?

In five half-years how many months?

Count by sixes to thirty.

Three cubes have how many corners?

Two pounds of candy will fill how many little bags, if there is an eighth of a pound in each bag?

Count by eights to twenty-four.

In a month of twenty-eight days how many weeks?

There are seven colors in a rainbow. If three rainbows were placed side by side, how many strips of color would there be? How many colors?

Count by sevens to twenty-eight.

Nine inches are a quarter of a yard. A half a yard then is how many inches? Three-quarters of a yard are how many inches?

Count by nines to twenty-seven.

Count by tens as far as you can.

Count by elevens as far as you can.

Two dozen oranges are how many oranges?

Two feet are how many inches?

In two years how many months?

Two cubes have how many edges?

One-half of a pint is how many gills?

One-half a working week is how many days?

One-half of an apple is how many eighths of an apple?

One-half of a dime is how many cents?

One-half of a foot is how many inches?

One-half of fourteen days is how many weeks?

One-half of a pound is how many ounces, if there are sixteen ounces in a pound?

There are eighteen inches in half a yard. If this ruler is half a yard long, how long is half the ruler?

One-half of a score of persons is how many persons?

One-half of twenty-two bonnets are how many bonnets?

One-half of a day is how many hours?

One-half of February is how many days?

One-half of a month which has thirty days is how many days?

One-half of fifty cents is how many cents?

One-third of a year is how many months? One-fourth of a year? One-sixth of a year?

One-third of a day is how many hours? One-fourth of a day? One-sixth of a day? One-eighth of a day?

One-fourth of a score is how many? One-fifth of a score?

One-fifth of a quarter of a dollar is how much?

One-fifth of a month is how many days?

Divide seven by four and tell me the answer. Divide nine by five. Divide sixteen by three; eighteen by four; twenty-one by ten; twenty-four by nine.

Take this number work:

$3 \times 7 =$	$8 \times 3 =$	$\frac{1}{5}$ of 30 =	$\frac{1}{5}$ of 30 =
$7 \times 3 =$	$6 \times 4 =$	$3 \times 9 =$	$\frac{1}{5}$ of 30 =
$21 \div 3 =$	$24 \div 6 =$	$27 \div 3 =$	$\frac{1}{5}$ of 30 =
$21 \div 7 =$	$24 \div 8 =$	$\frac{1}{5}$ of 27 =	$\frac{1}{5}$ of 30 =
$\frac{1}{5}$ of 21 =	$5 \times 5 =$	$4 \times 7 =$	$6 \times 5 =$
$2 \times 11 =$	$\frac{1}{5}$ of 25 =	$28 \div 4 =$	$30 \div 6 =$
$12 \times 2 =$	$\frac{1}{5}$ of 20 =	$\frac{1}{5}$ of 28 =	$30 \div 5 =$

Name the sum of each couple of numbers as I point.

7	4	8	5	9	3	11	4	8	8	7
6	8	5	6	4	10	3	7	7	6	5

See if you can add by couples of numbers when there are several numbers to be added, as in this case: 3

When I point to 5, think of the sum of 2 and 5, or 4
 7. When I point to 6, think of the sum of 2 and 6, or 5
 8, and say, "7 and 8 are 15." When I point to 5, say, 4
 "And 9 are 24." When I point to 3, say, "And 7 are 6
 31." 2

When I point this time, try if you can add quickly 5
 by the couples of numbers, thus: 7, 15, 24, 31. 2

Add these numbers in the same way:

2	5	6	2	10	6	5	6	7
7	2	4	5	4	3	5	6	3
3	6	2	4	3	8	7	5	5
5	1	10	5	4	10	7	3	7

United States Money.

I have two dollars in my purse. Who will express on the board what I have in my purse? (2 dollars.) You are right, but I can express it in another way: \$2.

Express: Three dollars. Seven dollars. Five dollars. Twenty dollars. Twenty-five dollars. Eight dollars. One hundred dollars. Sixty dollars. Forty-seven dollars. Eighty-six dollars.

Read what you have expressed.

What have I expressed? (\$5.) I wish to express five dollars and twenty-five cents. Can you show me how? Ned has it right. We always put the dot between the dollars and cents so the cents shall not get into dollars' place. The dollars are in the highest class, and the little cents don't belong with them.

Read what I have expressed:

\$6.25	\$7.84	\$9.63	\$12.44	\$73.82
\$3.50	\$2.04	\$8.02	\$75.08	\$60.10

Express: Three dollars and twenty-six cents.

Eight dollars and sixty cents.

Twelve dollars and a half.

Sixteen dollars and a quarter.

Thirty dollars and ten cents.

Three dollars and five cents.

Five dollars and three cents.

Twenty dollars and two cents.

Thirty dollars and three cents.

Fifty dollars and five cents.

Dry Measures.

Who has seen wooden measures like these somewhere else? What have you seen measured in any one of them? How much does the smallest measure hold? How much

does the next larger one hold? How many times as much as the first measure will the quart measure hold?

Who knows what the largest measure is called? It is a *peck* measure. If you buy this largest measure full of beans, what quantity will you buy? Name some articles that are sold by the peck.

I have here some beans. Who would like to find how many quarts of beans the peck measure holds? Willie may measure, and Annie may put a mark on the board for every quart he puts into the peck measure.

How many quarts does it take to make a peck?

If a horse eats a half of a peck of oats at a meal, how many quarts of oats does he eat?

If I pick a peck and a half of blueberries, how many quarts of blueberries do I pick?

If you buy two pecks of oats, how many quarts of oats do you buy?

A party of boys went chestnutting and found two pecks and a half. How many quarts of chestnuts did they find?

In three pecks how many quarts?

If the peck measure is a quarter full, how many more quarts will it hold? If it is three-quarters full, how many more quarts will it hold?

If the peck measure is five-eighths full, how many quarts are in the measure?

Express: Two pints are one quart.

Eight quarts are one peck.

In what two ways can you express on the board the addition of twenty and thirty?

In what two ways can you express the subtraction of twenty from thirty?

In what two ways can you express two times thirty?

In what one way can you express thirty divided by six?
 We have another way for this: $6 \overline{)30}$

5

You may express in this new way:

Thirty divided by six.

Twenty divided by five.

Twenty-seven divided by nine.

Twenty-eight divided by seven.

Twenty-eight divided by four.

Twenty-five divided by five.

Twenty-four divided by six.

Twenty-four divided by eight.

Twenty-one divided by seven.

Read what you have expressed.

Take this number work:

$4 \overline{)16}$

$5 \overline{)30}$

$4 \overline{)28}$

$12 \overline{)24}$

$6 \overline{)18}$

$7 \overline{)28}$

$6 \overline{)24}$

$11 \overline{)22}$

$7 \overline{)14}$

$8 \overline{)24}$

$3 \overline{)27}$

$10 \overline{)50}$

CHAPTER XXVIII.

FIRST STEPS IN WRITTEN ARITHMETIC.

§ 50. ADDITION.

When the sum of the units does not exceed nine.

Three men pastured their sheep in the same lot. One man had thirteen sheep in the lot, another had twenty-one, and the other had thirty-four. I want you to find how many sheep were in the lot. Show me with the splints the number thirteen; below it twenty-one, and below the other two numbers put thirty-four. In thirteen how many ones besides the one ten? In twenty-one how many besides the two tens? In thirty-four how many ones besides the three tens? How many ones altogether? (8.) How many tens in thirteen? in twenty-one? in thirty-four? How many tens altogether? Six tens and eight are how many? Then how many sheep were in the pasture?

NOTE. Let the numbers be shown in tens and ones, the tens being each in a bundle by itself.

Let examples be done in this way until the children can perform the operations in the right order without any suggestions from the teacher. When they can do this, they are ready to transfer their work to the board.

Who can express this example on the board? Willie may express it. All may express it.

Who would like to try one at the board and not use the splints? Those who cannot find the answer on the board may use the splints.

Take this example: A merchant had three pieces of cloth. In one piece were fifteen yards, in another thirty-one, and in the other twenty-three. How many yards of cloth in the three pieces?

Take this: I spent fourteen cents for some sugar, forty-two cents for some tea, and thirty cents for some butter. How many cents did I spend?

Take this: How many hours in a day and a half and ten hours more?

Find the sum of twelve, thirteen, fourteen, and fifty.

Find the sum of twenty-three, twenty, thirty-three, and thirteen.

Find the sum of forty-two, twenty-one, fourteen, and twenty-two.

Find the sum :

32	43	33	25	31	60
21	13	22	30	40	19
23	10	11	10	18	10
12	11	12	13	10	30
10	20	21	11	20	20
—	—	—	—	—	—
43	61	37	55	72	91
25	33	40	31	81	20
31	42	51	60	90	65
50	20	30	81	63	73
70	71	20	70	51	50
—	—	—	—	—	—

§ 51. SUBTRACTION.

When the number in each order of the minuend is greater than the corresponding number in the subtrahend.

I had fifty-seven chickens and sold twenty-four. I want you to find how many chickens I had left. Show me with

the splints the number I had before I sold any. Take out the ones I sold. Take out the tens I sold. How many did I have left?

NOTE. Let the pupils solve many similar problems, and always insist that the difference between the *ones* be found first. When the answer to any example of this kind can be readily determined through the use of objects, let the examples be performed at the board.

Express this example on the board: A man made a journey of sixty-eight miles on horseback. When he had gone forty-five miles, how much farther had he to go?

What number will you subtract first? What next? What is the answer?

Express these examples and find the answers:

A train of cars is seventy-nine minutes going from one station to another. If the train is in motion only sixty-seven minutes, how much time is taken in making stops?

A man who had ninety-six acres in his farm sold twenty-one acres. How many acres did his farm then contain.

There were forty-seven trees in an orchard, but fourteen were cut down. How many remained standing?

Find the difference between seventy-eight and thirty-five.

Find the difference between eighty-nine and fifty-four.

Find the difference between ninety-nine and sixty-six.

Find the difference between eighty-seven and thirty-three.

Find the difference:

98	87	75	69	58	87
— 43	— 34	— 32	— 25	— 15	— 53
—	—	—	—	—	—

99	88	77	97	89	86
— 65	— 53	— 34	— 62	— 66	— 22
—	—	—	—	—	—

§ 52. MULTIPLICATION.

When the multiplier is less than ten, and the product of any order in the multiplicand (unless it be the highest) is less than ten.

It takes twenty-one buttons for one jacket. I want to find how many buttons it will take for four jackets; you may help me. Show me first how many *ones* are required for four jackets, since it takes one for each jacket. Show me next how many *tens* are required for four jackets, since it takes two tens for each jacket. How many buttons in all?

There were two dozen buttons on each card; I want you to find how many there were on two cards. First show how many ones on the two cards, since there were four on each card. Next show how many tens on two cards, since there were two tens on each card. How many buttons on both cards?

If I have two quires of paper, how many sheets of paper do I have.

How many days are there in four months of thirty-one days each?

There are thirty-two quarts in a bushel. How many quarts in four bushels?

If you go to school twenty-one days in a month, how many days do you go in nine months?

[Give similar examples, and let the product be found with the objects until the pupils can do the work readily. Always require the product of the ones found first. Pass to board work when the objects are no longer needed.]

CHAPTER XXIX.

NUMBERS FROM THIRTY TO FIFTY.

§ 53. THE NUMBER THIRTY-TWO.

Facts to be taught:

8×4	$\frac{1}{4}$ of 32	$32 \div 8$
$32 \div 4$	4×8	$\frac{1}{4}$ of 32

NOTE. Teach each fact by itself, and follow the order indicated above.

Express the work in these examples on the board:—

If I have eight dollars in quarters, how many quarters have I? ($8 \times 4 = 32$.)

Eight gallons are how many quarts? ($8 \times 4 = 32$.)

Thirty-two quarts will fill how many half-peck measures? ($32 \div 4 = 8$.)

I bought 8 pounds of meal for thirty-two cents, what was the meal a pound? ($\frac{1}{4}$ of $32 = 4$.)

If eight horses require thirty-two shoes, how many shoes does each horse require? ($\frac{1}{8}$ of $32 = 4$.)

Four pies each cut into eighths will make how many pieces? ($4 \times 8 = 32$.)

If a man bought thirty-two quarts of cranberries, how many pecks did he buy? ($32 \div 8 = 4$.)

Thirty-two ox-shoes will shoe how many oxen all round? ($32 \div 8 = 4$.)

How many feet in circumference is a wheel which turns around four times in going over thirty-two feet? ($\frac{1}{4}$ of $32 = 8$.)

Ounces in a Pound.

Lift the larger weight. What does it weigh? (A pound.) Find a stone on this table which you think weighs a pound. Test its weight by these scales. Were you right? •Try again. Each mention something in this room which weighs a pound.

Mention some things that are bought and sold by the pound.

I have here some smaller weights. What will each of these weigh? (An ounce.) Here is some salt. Put in this paper what you think will weigh an ounce. When the paper of salt feels as heavy as the ounce weight, weigh it and see if it is more or less than an ounce.

Mention some things that are sold by the ounce.

Put the pound weight in one scale-pan, and as many ounces in the other as are required to balance the pound weight.

How many ounces equal a pound?

Express: Sixteen ounces equal one pound.

If I bought four ounces of potassium, what part of a pound did I buy?

If I bought eight ounces of gum arabic, what part of a pound did I buy?

If I bought a fourth of a pound of tea, how many ounces of tea did I buy?

If you bought half of a pound of cheese, how many ounces of cheese did you buy?

I ordered a pound of steak, but the piece brought me weighed a pound and a quarter. How many ounces did it weigh?

I bought a pound and a half of dates. How many ounces did the dates weigh?

I bought two pounds of chocolate. How many ounces did I buy?

§ 54. THE NUMBER THIRTY-FIVE.

Facts to be taught :

7×5	$\frac{1}{7}$ of 35	$35 \div 7$
$35 \div 5$	5×7	$\frac{1}{5}$ of 35

Express on the board :

In seven weeks how many school days? ($7 \times 5 = 35$.)

Seven ounces of potassium at five cents an ounce will cost how much? ($7 \times 5 = 35$.)

For thirty-five cents how many five-cent bunches of envelopes can I buy? ($35 \div 5 = 7$.)

Thirty-five ounces of liquid will fill how many vials, each containing five ounces? ($35 \div 5 = 7$.)

If a boy earns thirty-five dollars in seven months, what does he average to earn a month? ($\frac{1}{7}$ of 35 = 5.)

If I pay thirty-five cents for seven yards of cambric, what is that a yard? ($\frac{1}{7}$ of 35 = 5.)

In five weeks how many days? ($5 \times 7 = 35$.)

In thirty-five days how many weeks? ($35 \div 7 = 5$.)

If I pay thirty-five dollars for five weeks' board, what do I pay by the week? ($\frac{1}{5}$ of 35 = 7.)

Exercise for Review.

Draw two lines of the same length parallel to each other and about an inch apart. Mark one **A** and the other **B**. Divide **A** into fourths and **B** into fifths. Since **A** and **B** are the same length, one-fifth of **B** will be what part of **A**? Compare one-fifth of **A** with one-fourth of **A**, and tell me which is larger.

Which is larger, one-fourth of an apple or one-fifth of the same apple? One-fourth of an orange or one-fifth of the same orange? One-fourth or one-fifth of a dollar?

Find the sum of :

One dollar and four cents.

Four dollars and eleven cents.

Two dollars and ten cents.

Three dollars and two cents.

Three dollars.

Find the sum of :

Four dollars and twenty-three cents.

Two dollars and one cent.

Five dollars and thirty cents.

Three dollars and twenty-five cents.

Five dollars and ten cents.

Find the difference between :

Eight dollars fifty-four cents, and three dollars thirty-two cents.

Nine dollars seventy-eight cents, and four dollars three cents.

Twenty-eight dollars forty cents, and seven dollars twenty cents.

Eighty-nine dollars sixty-eight cents, and forty-seven dollars thirty-five cents.

Seventy-eight dollars ninety-nine cents, and fifty-three dollars twenty-seven cents.

Write the table of Liquid Measure.

Write the table of twos.

Who can write the table of threes?

Two ounces is what part of a pound?

Two pounds are how many ounces?

Four pecks and a half of currants are how many quarts of currants?

Two scores of years are how many years?

Three reams of paper are how many quires of paper?

Four twenty-dollar gold pieces are how much money?

Four twenty-five cent pieces are how much money?

In seventeen, how many tens, and how many ones besides?

In thirteen, how many tens, and how many ones besides?

In fifteen? In nineteen? In twenty-four? In twenty-

seven? In forty? In fifty-three? In seventy-five?

§ 55. THE NUMBER THIRTY-SIX.

Facts to be taught :

12×3	6×6	$\frac{1}{4}$ of 36
$36 \div 3$	$36 \div 6$	3×12
9×4	$\frac{1}{3}$ of 36	$36 \div 12$
$36 \div 4$	4×9	$\frac{1}{3}$ of 36
$\frac{1}{3}$ of 36	$36 \div 9$	$\frac{1}{2}$ of 36

Express on the board :

A side of our room is twelve yards long. How many feet in length is it? ($12 \times 3 = 36$.)

I bought three dozen oranges this morning. How many oranges did I buy? ($3 \times 12 = 36$.)

I have thirty-six cents in three-cent pieces. How many three-cent pieces have I? ($36 \div 3 = 12$.)

I have here a stick which is thirty-six inches long. How many feet in length is the stick? ($36 \div 12 = 3$.)

Nine gallon-jugs will hold how many quarts of vinegar? ($9 \times 4 = 36$.)

At nine cents a quarter, what will a pound of candy cost? ($4 \times 9 = 36$.)

Thirty-six panes of glass will supply how many windows, if it takes four panes for each window? ($36 \div 4 = 9$.)

At nine cents a quire, how many quires of paper can I buy for thirty-six cents? ($36 \div 9 = 4$.)

Six melons, each cut into sixths, will make how many pieces? ($6 \times 6 = 36$.)

Thirty-six eggs will make how many loaves of cake, if a half dozen is required for each loaf? ($36 \div 6 = 6$.)

If you divide a yard of ribbon into three equal parts, how many inches in length is each part? ($\frac{1}{3}$ of $36 = 12$.)

Thirty-six pounds of starch were put into four equal packages. How many pounds were in each package? ($\frac{1}{4}$ of $36 = 9$.)

If six yards of cloth cost thirty-six cents, what is the cloth a yard? ($\frac{1}{6}$ of $36 = 6$.)

If I can buy nine dozen clothes-pins for thirty-six cents, what are they worth a dozen? ($\frac{1}{9}$ of $36 = 4$.)

Exercise for Review.

Some silk, some ribbon, and some ruching together cost thirty-six cents. The silk and the ribbon cost twenty-four cents. The ribbon and the ruching cost eighteen cents. What did each of the articles cost? (Ruching 12 cents, ribbon 6 cents, silk 18 cents.)

We have a flight of stairs consisting of twenty steps. Each step is twelve inches broad and six inches deep. How many yards of carpeting will be required to carpet the stairs? (10 yards.)

How many plants, a foot apart, can be set in two borders, each five yards long? (32.) In three borders, each three yards long? (30.)

If in a winter's day it is dark from half-past four until half-past seven the next morning, how many hours is it from daylight to dark? (9 hours.)

Eddie had a ten-cent piece, three five-cent pieces, a three-cent, and a two-cent piece. He bought a kite for ten cents, and three oranges at five cents apiece. How many cents had he left? (5 cents.)

If a sheet of paper is doubled four times, how many leaves are made? (16.)

A long strip of lead was cut into five strips, each six inches long, and there were three inches of the lead left. How long was the strip of lead? (33 inches.)

A boy who was selling lilies sold five for two cents. How much money did he receive for thirty lilies? (12 cents.) For fifty lilies? (20 cents.)

If two men can lay a wall in thirty days, how long will it take four men to lay the wall? (15 days.) How long will it take six men? (10 days.)

I bought some oranges and some candy for twenty-four cents. The oranges cost twice as much as the candy. How much did the candy cost? (8 cents.)

I have a half of a dollar and John has a quarter of a dollar. What part of a dollar have we together? ($\frac{3}{4}$.)

It will take a fourth of a yard of velvet to face my hat, an eighth of a yard to bind it, and a half of a yard for bows. How much velvet do I need for the hat? ($\frac{7}{8}$.)

It takes a quarter of a yard of satin to make a bag, and an eighth of a yard to face it. How much satin will it take all together? ($\frac{3}{8}$.) What will the satin cost at eighty cents a yard? (30 cents.)

Pecks in a Bushel.

Look at these two measures. What do we call the smaller one? (A peck measure.)

The larger one holds a half-bushel. Measure these oats, and find how many pecks make a half-bushel.

Who can tell how many pecks make a whole bushel?

Write: Four pecks equal a bushel.

What is bought by the bushel?

A half-bushel of potatoes are how many pecks of potatoes?

A bushel of peas are how many pecks of peas?
 A bushel of apples are how many pecks of apples?
 A bushel of nuts are how many pecks of nuts?
 Four bushels are how many pecks?
 Six bushels are how many pecks?
 Nine bushels are how many pecks?
 Eight bushels are how many pecks?
 Seven bushels are how many pecks?
 Ten bushels are how many pecks?
 Five bushels are how many pecks?

§ 56. THE NUMBER FORTY.

Facts to be taught:

$\frac{1}{10}$ of 40	$40 \div 5$	5×8	$\frac{1}{5}$ of 40
8×5	$\frac{1}{8}$ of 40	$40 \div 8$	$\frac{1}{2}$ of 40

Express on the board:

If I can buy ten spools of thread for forty cents, what does one spool cost? ($\frac{1}{10}$ of 40 = 4.)

Eight gloves have how many fingers? ($8 \times 5 = 40$.)

How many dozen peaches, at five cents a dozen, can be bought for forty cents? ($40 \div 5 = 8$.)

If a boy earned forty dollars in eight weeks, how much did he earn in one week? ($\frac{1}{8}$ of 40 = 5.)

If a man works eight hours a day, how many hours will he work in five days? ($5 \times 8 = 40$.)

If a bicycle will go a mile in eight minutes, how many miles will it go in forty minutes? ($40 \div 8 = 5$.)

Susie recites forty lessons a week if she is at school five days in the week. How many lessons does she recite each day? ($\frac{1}{5}$ of 40 = 8.)

I bought forty yards of carpeting, and used one-half of it. How much did I use? ($\frac{1}{2}$ of 40 = 20.)

§ 57. THE NUMBER FORTY-TWO.

Facts to be taught:

7×6	$\frac{1}{7}$ of 42	$42 \div 7$
$42 \div 6$	6×7	$\frac{1}{6}$ of 42

Express on the board:

How many working days are there in seven weeks?
 $(7 \times 6 = 42.)$

If eggs are packed in layers of half a dozen each, how many layers will forty-two eggs make? $(42 \div 6 = 7.)$

If seven cans will hold forty-two quarts of milk, what will one can hold? $(\frac{1}{7}$ of 42 = 6.)

In six weeks how many days? $(6 \times 7 = 42.)$

If we use a bag of flour a week, how many bags of flour do we use in forty-two days? $(42 \div 7 = 6.)$

A little boy who works in a mill earns forty-two cents a week. How much does he earn a day? $(\frac{1}{7}$ of 42 = 6.)

§ 58. THE NUMBER FORTY-FIVE.

Facts to be taught:

9×5	$\frac{1}{9}$ of 45	$45 \div 9$
$45 \div 5$	5×9	$\frac{1}{5}$ of 45

Express on the board:

If a man can paint five pails in an hour, how many pails can he paint in nine hours? $(9 \times 5 = 45.)$

How many weeks will forty-five pounds of butter last a family when five pounds are used a week? $(45 \div 5 = 9.)$

If it takes forty-five pickets for nine gates, how many pickets will it take for one gate? $(\frac{1}{9}$ of 45 = 5.)

A man set out cabbage plants, nine in a row. How many were there in five rows? $(5 \times 9 = 45.)$

If a dozen silver forks cost nine dollars, how many dozen can be bought for forty-five dollars? $(45 \div 9 = 5.)$

If a newsboy sells forty-five papers, and one-fifth are weeklies and the rest dailies, how many weeklies does he sell? ($\frac{1}{5}$ of 45 = 9.) How many dailies? ($45 - 9 = 36$.)

§ 59. FIRST STEP IN MULTIPLICATION OF FRACTIONS.

If it takes one-third of a yard of lace to trim one sleeve, how many thirds will be required for a pair of sleeves? for two pairs of sleeves?

If a dozen bananas cost a quarter of a dollar, how many quarters will six dozen bananas cost?

If John earns two-fifths of a dollar a day, how many fifths will he earn in a week?

It is three-eighths of a mile to school. If I go over the distance four times a day, how many eighths of a mile shall I walk in a day?

A large family used five-sixths of a pound of sugar every night at supper. How many sixths did the family use in a week? Express this example on the board.

NOTE. If the pupil has any difficulty in expressing examples in which fractions occur, require him to express a similar example in which whole numbers occur, and he will perceive the analogy.

Find the products:

$$\begin{array}{llll}
 2 \times \frac{3}{8} = \frac{6}{8} & 2 \times \frac{4}{9} = \frac{8}{9} & 3 \times \frac{3}{10} = \frac{9}{10} & 3 \times \frac{4}{18} = \frac{12}{18} \\
 4 \times \frac{3}{18} = \frac{12}{18} & 2 \times \frac{5}{11} = \frac{10}{11} & 3 \times \frac{7}{2} = \frac{21}{2} & 3 \times \frac{5}{16} = \frac{15}{16} \\
 3 \times \frac{2}{9} = \frac{6}{9} & 3 \times \frac{3}{11} = \frac{9}{11} & 4 \times \frac{5}{21} = \frac{20}{21} & 4 \times \frac{4}{17} = \frac{16}{17}
 \end{array}$$

§ 60. THE NUMBER FORTY-EIGHT.

Facts to be taught:

$$\begin{array}{lll}
 12 \times 4 & 48 \div 6 & \frac{1}{6} \text{ of } 48 \\
 48 \div 4 & \frac{1}{8} \text{ of } 48 & 4 \times 12 \\
 \frac{1}{12} \text{ of } 48 & 6 \times 8 & 48 \div 12 \\
 8 \times 6 & 48 \div 8 & \frac{1}{4} \text{ of } 48
 \end{array}$$

Express on the board :

Twelve chairs have how many legs? ($12 \times 4 = 48$.)

If it takes four rails for a gate, how many gates will forty-eight rails make? ($48 \div 4 = 12$.)

Forty-eight books were put into twelve equal piles. How many books were in each pile? ($\frac{1}{12}$ of $48 = 4$.)

If one cake of maple sugar weighs six pounds, what will eight cakes weigh? ($8 \times 6 = 48$.)

If six baskets of peaches together hold a peck of peaches, how many pecks of peaches will forty-eight baskets hold? ($48 \div 6 = 8$.)

If eight bunches of envelopes cost forty-eight cents, what does one bunch cost? ($\frac{1}{8}$ of $48 = 6$.)

Six oxen wear how many shoes all together? ($6 \times 8 = 48$.)

How many pecks in forty-eight quarts? ($48 \div 8 = 6$.)

If a boy earns forty-eight cents in six hours, what is that an hour? ($\frac{1}{6}$ of $48 = 8$.)

Four dozen oranges are how many oranges? ($4 \times 12 = 48$.)

In forty-eight inches how many feet? ($48 \div 12 = 4$.)

Forty-eight bunches of matches were put into four equal packages. How many bunches were in each package? ($\frac{1}{4}$ of $48 = 12$.)

§ 61. THE NUMBER FORTY-NINE.

Facts to be taught :

$$7 \times 7$$

$$49 \div 7$$

$$\frac{1}{7} \text{ of } 49$$

Express on the board :

If I put seven melon seeds in a hill, how many seeds do I put in seven hills? ($7 \times 7 = 49$.)

A cooper has forty-nine feet of hoop iron. If it takes

seven feet for one hoop, how many hoops can he make?
($49 \div 7 = 7$.)

If seven pounds of sugar cost forty-nine cents, what does one pound cost? ($\frac{1}{7}$ of $49 = 7$.)

Exercise for Review.

REMARK. Application of numbers in examples like the following is an important part of the work in Arithmetic. It is not expected that the pupil will solve the problems by himself. He will need to be led by appropriate questions to reason out the result, or by illustration to see the result. Each example is a teaching exercise in itself, and the whole exercise is calculated to teach the pupil how to apply his knowledge of numbers. The exercise is a test exercise only in the sense that it tests knowledge of facts. The examples are not too difficult for this grade of work.

A man who earned thirty dollars a week paid ten dollars a week for board. Another man earned but twenty-five dollars a week, and paid four dollars a week for board. Which one was better off at the end of the week?

A man counting his eggs for market found he had half a hundred. He kept two for his own use, and packed the rest in layers of half a dozen each. How many layers had he?

John earns three dollars a week. If he spends one dollar one week, three dollars the next week, and five the next week, how much money has he left from his three weeks' earnings? What are his average expenses a week?

If I spend ten dollars one week, twelve dollars the next week, and eleven dollars the next week, what are my average expenses per week?

Nellie had thirty pretty buttons and Elsie had ten buttons. Nellie agreed to give Elsie enough buttons so that each should have the same number if Elsie would exchange some of her buttons for Nellie's. How many buttons did Nellie have after she had done as she agreed?

A garden plat nine feet by six feet was bordered by a walk two feet wide. How much farther was it around the outside of the walk than around the garden plat?

While out shopping this morning I spent half the money in my purse. I had five dollars when I returned home. How much money did I spend?

After spending a fourth of her money Annie had twelve cents left. How many cents had she at first?

After Harry had done a third of his examples he had eighteen more to do. How many had he to do all together?

A little girl's car-fare was fifteen cents, and her mamma's was twice as much. How much was the fare for both?

John guessed that a pumpkin weighed ten pounds and four ounces. George guessed it weighed ten pounds and ten ounces. Neither was right, but one was just as nearly right as the other. How much did the pumpkin weigh?

After spending a quarter of a dollar I had three times as much money in my purse as I spent. How much money had I at first?

A wood-sawyer had sticks of wood four feet long which he was to cut into sticks a foot long. How many cuts did he make in sawing a dozen sticks of wood? How many sticks did he make?

I bought eight yards of cloth at six cents a yard, and gave in payment a quarter of a dollar and three ten-cent pieces. How much change ought I to receive?

Susie is fifteen years old and her brother is ten years old. How old was Susie when twice as old as her brother?

Johnnie is five years old and his father is thirty years old. How old will Johnnie be when half as old as his father?

James and his sister went berrying one day. At night, after their berries were picked over, they turned them into some measures to see how many they had. The berries

filled a half-bushel measure, a peck measure, a two-quart measure, a quart measure, and a pint measure. How many quarts of berries had they?

If there was an animal whose number of eyes doubled each year he lived, and who had two eyes to begin with, how many eyes had he in five years?

I got a hundred dollars from the bank in six bills. Two of the bills were ten dollars each. What was each of the other bills if they were each for the same amount?

A man spent six-eighths of what he earned. What part of his earnings had he left? If he had spent seven-ninths of his earnings, what part would he have had left? If he had spent six-sevenths, what part would he have had left?

Two boats are fifty miles apart, and are sailing toward each other. One sails at the rate of six miles an hour, and the other at the rate of four miles an hour. In how many hours will they pass each other?

Two boats are sailing in the same direction and are ten miles apart, but the one that is behind goes two miles an hour faster than the one ahead. How many hours will it take the boat that is behind to make up the ten miles, and so catch up with the other boat?

One man earns two hundred dollars a month, but has work only half of the year. Another man earns a hundred dollars a month, and works every month in the year. How do the earnings of the two men for a year compare?

How many strokes does a clock give in twelve hours?

We have a clock which strikes every hour, and strikes one at quarter past, two at half-past, and three at quarter of the hour. How many strokes does such a clock give from five minutes past twelve to five minutes past six?

How many yards of carpeting a yard wide will be needed to carpet a room five yards long and five yards wide?

How many yards of carpeting a yard wide will it take

to carpet a hallway that is a yard and a half wide and six yards long?

How many yards of cloth half a yard wide will it take to line a box that is a yard long, a half a yard wide, and a half a yard deep?

If a wax-plant doubled its number of blossoms each year, and had two blossoms the first year, how many blossoms had it the seventh year?

If you divide a line into two parts, so that one part shall be twice as large as the other, what fraction of the whole line is each part?

After dividing thirteen by four, what part of four shall I have left?

After dividing fifteen by four, what part of four shall I have left?

After dividing seventeen by three, what part of three shall I have left?

After dividing nineteen by six, what part of six shall I have left? after dividing twenty by six? twenty-two by six? twenty-five by six?

If I divide two apples equally among four boys, what part of an apple shall I give to each?

If you and Susie and I had each three-quarters of a dollar, how many dollars should we have together?

If a boy earned three-quarters of a dollar a day, how much would he earn in a week?

If it takes two-fifths of a yard of ribbon for a bow, how much ribbon will be needed for five bows? for ten bows?

If you put a bushel of apples into four baskets, each holding the same quantity, how many quarts does each basket hold?

My album will hold four pictures on a page. How many pictures will the album hold if there are six leaves in it?

If it takes forty yards of carpeting a yard wide for a room five yards wide, how long is the room?

If sugar that cost seven cents a pound is sold at ten cents a pound, how much is gained on thirteen pounds?

If the material for a dress costs ten dollars, and the trimmings and making cost twice as much, how much does the dress cost?

A man bought an old table for five dollars. He paid two dollars for having it scraped, and sold it for ten dollars. How much did he gain?

A farmer's wife sold four dozen eggs at a quarter of a dollar a dozen, and took her pay in cotton cloth at ten cents a yard. How many yards did she get?

My watch will run from eight o'clock in the morning until twelve o'clock at night the next day. How many hours will my watch run?

If I leave the school-house at five o'clock at night, and am there at half-past eight in the morning, how many hours from the time I leave the school-house until I return?

If a locomotive goes at the rate of two miles in three minutes, how far will it go in fifteen minutes? in twenty-one minutes? in eighteen minutes? in thirty minutes? At the same rate, how long will it take the locomotive to go four miles? eight miles? sixteen miles?

If you make a little book by doubling a piece of paper six times, how many leaves will your book have? How many pages?

If there are fifty pages in a book, what are the two middle pages numbered? If there are fifty leaves in a book, what are the two middle pages numbered?

A boy earned four dollars a week. How much money had he at the end of twelve weeks if he had spent two weeks' earnings?

Four cubes have how many faces? have how many corners? have how many edges?

At the rate of a half-cent for every ounce, how much will it cost to send a package weighing two pounds?

Nellie is three years old the eleventh of September; Johnnie is six years old the seventh of the same month. How much older than Nellie is Johnnie?

Ida is nine years old the ninth of May; Jennie is seventeen years old the ninth of September. How much older than Ida is Jennie?

A man saved thirty-two dollars one month, half as much the next month, and spent eight dollars more than he earned the next month. How much money had he at the end of the three months?

A boy who earned three dollars a week spent two dollars one week, one dollar another week, and a half of a dollar each of two other weeks. How much had he saved at the end of four weeks?

If I draw as large a square as I can in a circle that is twelve inches across, how many inches will it be across the square from corner to corner?

How many square tiles, six inches on a side, can I place around the outside of a hearth that is four feet long and two feet deep? Represent the hearth on the board, and see if you are right. If the tiles were only four inches on a side, how many could I place around the outside?

Henry's hoop is six feet round the outside. If it turns round eight times without tumbling over, how much ground does it pass over?

How many desks, each occupying two feet of room, and placed two feet apart, can be set across a room that is thirty-six feet wide, if the two end desks are each three feet from the wall?

CHAPTER XXX.

PROCESSES IN WRITTEN ARITHMETIC CONTINUED.

§ 62. SECOND STEP IN ADDITION.

When the sum of the ones exceeds nine.

Three boys were counting their money. One boy had a quarter of a dollar, another boy had ten cents more, and the third boy had thirty-eight cents. Let us see how much money they had all together. Show me each of the numbers. What will you do first, Willie? (Find the sum of the ones.) How many ones? (Eighteen.) In eighteen how many tens? Put this rubber band about the ten. Who thinks what to do with this ten? (Put it with our other tens.) How many tens will you then have? How much money had the three boys together?

A farmer sent to market a dozen and a half bushels of potatoes at one time, two dozen bushels at another time, and three dozen bushels at another time. Let us find how many bushels he sent all together. Of what will you find the sum first, Eddie? (Of the ones.) How many ones? How many tens in *twenty-two*? Put a rubber band about each ten. What will you do with these two tens, Annie? How many tens will you then have? How many bushels of potatoes did the farmer send to market?

NOTE. As soon as the children can readily find the answers to such examples as the above by means of the objects, send them to the board to perform similar examples. Give much practice.

Other steps in addition :

736	1436
254	3798
76	2764
39	1978
<hr/>	<hr/>

§ 63. SECOND STEP IN SUBTRACTION.

When the number in the right-hand order of the minuend is less than the number in the corresponding order of the subtrahend.

There were fifty-two children in a room, but a class of twenty-eight was excused. We want to find how many were left in the room. Show me the number in the room before any left the room. Subtract the number of ones first. How many ones must you subtract? (8.) Who knows where to look for more ones? (Break a bundle of tens.) Yes, break a bundle of tens and put the ten with the ones. Now subtract eight. How many ones have you left? (4.) How many tens must you subtract? (2.) Subtracting the two tens, what number have you left? (24.) How many children then remained in the room?

NOTE. Give similar examples for work with objects until the pupils can readily find the answer. Take care to have the work performed in the right order, viz.: First. Take a ten and unite it with the ones of the minuend. Second. Subtract the given number of ones. Third. Subtract the tens.

Preparatory to passing from work with the objects to work with figures, question the pupil in regard to the order he follows, thus: In finding the difference between seventy-six and thirty-seven, what do you do first? (Take a ten and put it with the ones.) What next? (Subtract seven.) What next? (Subtract three tens.)

Take this example at the board :

A man who had eighty-three chickens sold fifty-nine. How many had he left?

(When the work is completed it will stand on the board in the form shown in the margin. After some practice the pupil will not need to express the "borrowing," but may carry on the work mentally.)

613
78
59
14

Other steps in subtraction :

350	702	6000
- 105	- 57	- 999
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§ 64. SECOND STEP IN MULTIPLICATION.

When the multiplier consists of one figure, and the product of the number in the lowest order of the multiplicand by the multiplier exceeds nine.

I had a square box that was sixteen inches on a side. I want you to tell me how far it was round the box. What will you find first? (Four sixes.) How many tens in your answer? Put a rubber band about each ten. What will you find next? (Four tens.) What will you do with the two tens you found in twenty-four? (Add them to the four tens.) How many inches round the box? (64.)

(Begin with work on the board as soon as the pupil comprehends that he is to separate the tens from the ones in the product of units by units, and that he is to add these tens to the product of the tens. It is well to have the partial products expressed separately at first, as shown in the work written in the margin. In a short time the pupil will learn to "carry" mentally, and express only the entire product.)

38
5

40
150

190

Find the products :

46	59	27	84
7	6	8	9
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CHAPTER XXXI.

NUMBERS FROM FIFTY TO SIXTY.

§ 65. THE NUMBER FIFTY-FOUR.

Facts to be taught :

9×6	$\frac{1}{9}$ of 54	$54 \div 9$
$54 \div 6$	6×9	$\frac{1}{6}$ of 54

§ 66. THE NUMBER FIFTY-SIX.

Facts to be taught :

8×7	$\frac{1}{8}$ of 56	$56 \div 8$
$56 \div 7$	7×8	$\frac{1}{7}$ of 56

§ 67. THE NUMBER SIXTY.

Facts to be taught :

12×5	5×12	$\frac{1}{2}$ of 60
$60 \div 5$	$60 \div 12$	$\frac{1}{6}$ of 60
$\frac{1}{12}$ of 60	$\frac{1}{5}$ of 60	$\frac{1}{10}$ of 60

Exercise for Review.

How many months in four years?

Counting four weeks and two days to a month, how many weeks are in a year?

A quire of paper is usually divided into four equal packages. How many sheets of paper are in each package? How many sheets of paper in eight such packages? How many quires?

A man bought six pecks of cranberries. If his family ate a quart a day, how many days did they last?

A cheese which weighed forty-eight pounds was cut into quarters. What did each quarter weigh? What did half of the cheese weigh?

What five equal pieces of money make fifty cents? What ten equal pieces? What two equal pieces? What twenty-five equal pieces? What fifty equal pieces?

Nine flies have how many legs all together?

If it takes a half-dozen eggs to make a loaf of cake, how many eggs will it take to make nine loaves?

If a man works at his trade nine hours a day, how many hours does he work in a week?

If I can buy a ticket and a half for six cents, how many tickets can I buy for fifty-four cents?

A roll of wall-paper fifty-four feet long will make how many strips, if our room is nine feet high?

A chamber-set was valued at fifty-four dollars, but, being damaged, the price was reduced one-sixth. What then was its price?

There were fifty-four pounds of butter in a tub. How much remained when one-ninth of it had been used?

How many school-days in eleven weeks?

If a star-fish has five arms, how many arms have eleven star-fish?

I bought five sticks of braid, each of which lacked one yard of containing a dozen yards. How many yards of braid were in the five sticks together?

Of fifty-five plants which the gardener set out this spring, one-fifth were killed by the frost. How many were killed?

How many days in seven weeks? in eight weeks?

I bought eight yards of cambric at seven cents a yard, and gave in payment a fifty-cent coin and a ten-cent coin. What change ought I to receive?

How long will seven pecks of oats last a horse if he eats seven quarts a day?

A man paid fifty-six cents for peaches, at eight cents a dozen. Two dozen decayed, and he sold the rest at twelve cents a dozen. Did he gain or lose, and how much?

A man bought a chamber-set for fifty-six dollars, and a rocking-chair for one-eighth as much. What did he pay for both together.

A man bought a cooking-range for fifty-six dollars, and a small oil-stove for one-seventh as much. What was the difference in price?

A man who sells shoe-strings at six cents a dozen gains a cent on each dozen? What did twelve dozen cost him?

In five feet how many inches?

In five years how many months?

In five dozen how many things?

If you can buy two apples for five cents, how many apples can you buy for sixty cents?

A newsboy buys eight papers for twelve cents. How many papers will he get for sixty cents? If he sells his papers for two cents apiece, what does he gain on eight papers? on sixteen papers? on twenty-four papers? How many dozen papers must he sell to gain sixty cents?

In sixty school-days how many school-weeks?

If it takes sixty tiles for a fire-place, how many dozen tiles does it take?

If your vacation is sixty days long, and you spend a fifth of the time at home, how many weeks are you at home?

What twelve equal numbers make sixty? What then is one-twelfth of sixty?

What piece of money is one-twelfth of sixty cents?

If in a term of twelve weeks a boy is absent one-sixth of the number of school-days, how many school-weeks is he absent?

When tea is sixty cents a pound, what will one-sixth of a pound cost? one-tenth of a pound? one-half of a pound?

My expenses for one month were sixty dollars. I spent one-half of the money for board, one-fifth for clothing, one-sixth for books and stationery, one-tenth for hired labor, and the rest in travelling. What did I spend for each item?

If a man buys old rubber at a half a cent a pound, what will he pay for sixty pounds? for fifty pounds? How much rubber will thirty cents buy? twenty-five cents?

How much will a man gain on four pounds of rubber if he buys it at a half a cent a pound, and makes it into toys which he sells at twenty cents a pound, if there is a waste of one pound?

If a loaf of bread costs the baker seven cents, and he sells it for ten cents, how much does he gain on a dozen loaves of bread?

A Second of Time.

What have I here? (A weight attached to a string.) The string is thirty-nine inches long. I will hang it on this hook, and make the weight swing. Whenever it passes just under the hook you may count.

How many have you counted? (10.)

Between each of your counts was a second of time. How many seconds was the weight swinging?

As I swing the weight again you may say aloud, but softly, *one second, two seconds, three seconds, and so on.* How many seconds did the weight swing this time? (15.)

You may now count silently, and when twenty seconds have passed, Eddie may stop the weight. Was he right, class?

When thirty seconds have passed, Annie may stop the weight.

When forty-five seconds have passed, Walter may stop the weight.

Count sixty seconds now, and then I will stop the weight.

You may see how many seconds it takes me to walk to the back of the room ; how many seconds it takes Minnie to make her slate clean ; how many seconds it takes Walter to write his name.

Annie may turn so that she cannot see the weight ; count ten seconds and turn back again. The others may count ten seconds by the swinging of the weight. Did Annie count right ? (She counted too fast.)

Emily may turn and count fifteen seconds. Was she right ? (She counted too fast.)

Walter may turn and count thirty seconds. Was Walter right ? (He was one second ahead.)

Half of the class may turn and count sixty seconds. The other half count by the swinging of the weight, and see who of those who turn are right.

All shut your eyes, and when you think sixty seconds have passed, you may raise hands. I will tell you who is nearest right.

Seconds in a Minute.

Look at the smallest hand of my watch. You may tell me when it has gone entirely round the little circle, and I will tell you how many swings the weight has made.

I will now watch the little hand, and you may count the swings. Do you find it to be sixty times, too ?

How many seconds then does it take the little hand to go entirely round the circle ?

Who knows what name we give to sixty seconds of time ? Carrie says it is a *minute* of time, and that is right.

Write on the board : Sixty seconds make a minute.

Who thinks he can tell when a minute of time is gone,

without looking at the watch or counting by the weight? We will let John try. He may step outside the door and wait just a minute. The others may see by the watch or by the weight how long he is gone. How long was he gone? (A half of a minute.)

Step to the board and write your names as many times as you can in a minute. When you have been writing a minute I will say, Now.

See how many different words you can write in a minute.

See how many words that rhyme you can write in a minute?

See how long a column of numbers you can write and add in a minute.

See if you can draw a square, an equilateral triangle, and a rectangle in a minute.

Make all the marks of punctuation, and all the letters of the alphabet in a minute if you can.

Count silently as high as you can in a minute.

How many seconds in half a minute? in one-fifth of a minute? in one-sixth of a minute? in one-tenth of a minute? in one-twelfth of a minute?

Ten seconds are what part of a minute?

Six seconds are what part of a minute?

Twelve seconds are what part of a minute?

Five seconds are what part of a minute?

Fifteen seconds are what part of a minute?

Twenty seconds are what part of a minute?

Minutes in an Hour.

Who knows how long it takes the long hand to go round the face of the clock? You are all right. It does take an hour.

Who knows how many minutes it takes the long hand to go from twelve to one? It does take five minutes. How

many minutes does it take the long hand to go from one to two? from two to three? from three to four? How many minutes does it take the long hand to go from any figure to the next figure? Let us see now if we can count up the number of minutes it takes the long hand to go round the face of the clock. If it takes five minutes to move from figure to figure, how many fives must we count? Twelve fives make how many? Then how many minutes does it take the long hand to go round the face of the clock? How many minutes then in an hour?

Write on the board : Sixty minutes make an hour.

How many minutes in half an hour? in one-quarter of an hour? in one-sixth of an hour? in one-twelfth of an hour? in one-third of an hour?

How many minutes in two hours? in three hours? in four hours?

How many minutes are you in school if the session is five hours?

If I stay an hour longer, how many minutes am I in school?

When the long hand is at one, how many minutes past the hour is it? When the long hand is at two, how many minutes past the hour is it? When it is at three? at four? at five? at six?

How many minutes of the hour is it when the long hand is at seven? at eight? at nine? at ten? at eleven?

At what figure does the long hand point at quarter past the hour? at half past the hour? at quarter of the hour? at twenty minutes past the hour? at twenty minutes of the hour?

Complete this table :

60 seconds =	7 days =
60 minutes =	4 weeks and 2 days =
24 hours =	12 months =

CHAPTER XXXII.

NUMBERS FROM SIXTY TO SEVENTY.

§ 68. THE NUMBER SIXTY-THREE.

Facts to be taught :

9×7	$\frac{1}{9}$ of 63	$63 \div 9$
$63 \div 7$	7×9	$\frac{1}{7}$ of 63

§ 69. THE NUMBER SIXTY-FOUR.

Facts to be taught :

8×8	$\frac{1}{8}$ of 64
$64 \div 8$	$\frac{1}{2}$ of 64

Exercise for Review.

In nine weeks how many days ?

In nine weeks how many working-days ?

In nine weeks how many school-days ?

A man pays three dollars a week for his room, and four dollars a week for board. What does he pay in nine weeks ?

If I pay a dollar a day for board, what do I pay for nine weeks' board ?

A farmer sold a dozen eggs for twenty-three cents, and five quarts of berries at eight cents a quart, and took in payment sugar at seven cents a pound. How many pounds did he receive ?

If it took a man sixty-three days to do a piece of work, how long did it take him to do one-ninth of the work ?

If I pay sixty-three dollars for nine weeks' board, what is that a day?

When bananas are five cents each, and oranges four cents, what must I pay for seven of each?

One room is four yards long, and another room five yards long. How many yards of carpeting will it take for both rooms if each takes seven strips?

Twenty-one yards of wall-paper will cut how many strips of paper each nine feet long?

A man who is three score and three years old is seven times as old as his grandson. How old is his grandson?

If a man earns sixty-three dollars a month, and spends fifty-six dollars, how many months will it take him to save a month's wages? What part of a year?

A stationer who buys blank-books for forty-one cents sells them at fifty cents each. If in one day he gains sixty-three cents from the sale of these books, how many books has he sold?

A boy who earns eight cents an hour works eight hours a day. How much does he earn in a day? If he spends fourteen cents a day, how much money will he have at the end of the week?

What must I pay for a peck of cherries at eight cents a quart? If they shrink one-half in cooking, how many pint jars of sauce shall I have? If cherries cost eleven cents a pint when canned, how much money do I save in putting up a peck of cherries for myself?

If a glazier has sixty-four panes of glass, how many windows each containing eight panes can he fill?

If it takes a steamer eight days to make a trip, how many trips will it make in two months of thirty-one days each, and two more days?

If sixty-four pounds of flour will last a family eight weeks, how much does the family use in one week?

A bicycle goes sixty-four miles in eight hours. What is the rate per hour?

There are thirty-two quarts in a bushel. How many quarts in two bushels?

A two-pound weight equals thirty-two ounces. How many ounces will a four-pound weight equal?

If a bushel of beans weighs sixty-four pounds, what will a half-bushel weigh? What will a peck weigh? What will a quart weigh?

If a pound of cake costs the baker thirty-five cents, and he sells it for forty cents, what does he gain on a dozen pounds of cake?

If a sick man must take two powders every three hours, how many powders does he take in a day, if he sleeps six hours? How many will he take in five days?

If a French coin called a franc is worth twenty cents of our money, how many francs are worth a dollar? Three dollars are equal to how many francs? Ten dollars are equal to how many francs?

If two kinds of beans worth eight cents and ten cents a quart are mixed, what will be the price of a quart of the mixture?

How many bottles will be required for nine gallons of wine, if it takes five bottles for a gallon?

Forty bottles will hold how many gallons of wine at five bottles for a gallon?

If a dealer in gloves buys at a dollar a pair, and sells at a dollar and a quarter, how much does he gain on a dozen pairs of gloves if one pair of gloves proves worthless?

If children's stockings can be bought at twelve and a half cents apiece, what will four pairs cost? What part of a dollar?

How many legs must be made for twenty-one wooden horses?

There are fifty-two weeks in a year. How many weeks are in four years?

There are thirty-two quarts in a bushel. How many quarts in three bushels?

If a bushel of potatoes weighs sixty pounds, what will nine bushels weigh?

A man who had ninety-seven turkeys sold seventy-four. How many had he left?

A barrel of flour weighs one hundred and ninety-six pounds. When sixty-four pounds have been used, how many pounds are left?

If there are eighty-seven pages in your reader, and you have read fifty-four pages, how many pages have you still to read?

What was the cost of a man's breakfast if he paid five cents for a cup of coffee, twelve cents for fish, and thirty-two cents for bread and meat?

A lady doing her marketing bought a head of lettuce for twenty-three cents, some tomatoes for fifteen cents, and some beef for fifty cents. How much did she spend?

A house three stories high had eight windows for each story in front, half that number in each story behind, and a fourth of that number on each side. How many windows had the house?

What is the height of a house if the ground floor is twelve feet high, the next floor eleven feet high, and the garrets and roof fifteen feet high?

Three times four-sixths are how many whole ones?

Four times five-tenths are how many whole ones?

Three times six-ninths are how many whole ones?

Four times six-eighths are how many whole ones?

Six times six-ninths are how many whole ones?

Twelve times four-sixths are how many whole ones?

Square Numbers.

Draw a square two inches on a side. Divide it into square inches. How many square inches does the square contain?

If the square were two feet on a side, how many square feet would it contain?

If the square were two yards on a side, how many square yards would it contain?

If any square is two units on a side, how many such square units will it contain?

Since any square two units on a side contains four square units, we say the square of two is four.

Write: The square of two is four.

We usually express it this short way: $2^2 = 4$.

Draw a square *three* inches on a side, and find the number of square inches it contains.

Draw a square three feet on a side, and find how many square feet it contains.

How many square yards in a square that is three yards on a side?

If a square is three units on a side, how many such square units will it contain?

What, then, is the square of three?

Express it in this way: $3^2 = 9$.

Who can think how many square inches a square four inches on a side will contain? Find out by drawing a square if you cannot think.

If any square is four units on a side, how many such square units will it contain?

Express the square of four. ($4^2 = 16$.)

If a square is five units on a side, how many square units will it contain?

Express the square of five. ($5^2 = 25$.)

If a square is six units on a side, how many square units will it contain?

Express the square of six. ($6^2 = 36$.)

Express the square of seven.

If a square is eight inches on a side, how many square inches will it contain?

Express the square of eight.

Name the numbers between one and twenty that are square numbers; between twenty and fifty. What other square numbers do you know?

Odd and Even Numbers.

Name a number that will contain two without a remainder. (4.)

Four is an *even* number.

Name another number that will contain two without a remainder. (8.)

Eight is an *even* number.

Name another even number.

Name as many even numbers as you can think of.

Begin with two and name them in order.

Who can write a dozen even numbers in a minute?

Write a score of numbers that are not even.

Such numbers are *odd*.

Name ten odd numbers, beginning with one.

See if you can discover with what figures even numbers must end.

If even numbers end with 0, 2, 4, 6, or 8, with what figures do odd numbers end?

As I express these numbers you may tell me whether they are odd or even: 15, 9, 18, 21, 25, 26, 30, 74, 87.

CHAPTER XXXIII.

NUMBERS FROM SEVENTY TO EIGHTY-FOUR.

§ 70. THE NUMBER SEVENTY-TWO.

Facts to be taught :

12×6	6×12	9×8	8×9
$72 \div 6$	$72 \div 12$	$72 \div 8$	$72 \div 9$
$\frac{1}{12}$ of 72	$\frac{1}{6}$ of 72	$\frac{1}{9}$ of 72	$\frac{1}{8}$ of 72

§ 71. THE NUMBER EIGHTY-ONE.

Facts to be taught :

9×9	$\frac{1}{9}$ of 81	$81 \div 9$	9^2
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Exercise for Review.

A man who buys chairs for twenty-four dollars per dozen sells them for thirty dollars per dozen. What does he gain on twelve dozen chairs?

If I pay a cent apiece for apples, two cents apiece for peaches, and three cents apiece for pears, what do I pay for a dozen of each?

A stationer bought twelve dozen pens for seventy-two cents, and sold them for twelve cents a dozen. What did he gain on the lot?

If a man saves from his earnings twelve dollars a month, how much will he save in half a year?

If I earn thirty-six dollars a week, and save one-sixth of it, in how many weeks shall I save seventy-two dollars?

Two ships are sailing in the same direction, seventy-two miles apart. If the second ship gains on the other twelve miles a day, in how many days will it overtake the first?

If the price of a mirror is seventy-two dollars, and one-sixth of the price is deducted from it, what then is the price of the mirror?

A man paid sixty-four dollars for chairs at eight dollars each, and sold them for nine dollars each. For how much did he sell them?

How many baskets will six dozen oranges fill, if each basket holds eight oranges?

A horse travelled seventy-two miles in twelve hours. A bicycle went the same distance in nine hours. What was the difference in speed per hour between the horse and the bicycle?

If a boy begins school at five years old and goes to school nine months a year until he is fourteen years old, how many months schooling will he have if he is out of school one year in the meantime?

Eight years give how many school months at nine school months a year?

Six dozen rosebuds will be enough for how many bouquets, if nine are used for each bouquet?

My room is six feet between the ceiling and the mop-board. If there is a bordering above and below, each one-eighth of this distance in width, how many inches between the two borders?

If it costs five cents for meat and four cents for bread for breakfast, what will nine breakfasts cost?

I have in my purse three quarters of a dollar, a five-cent piece, and one cent. How many slates at nine cents each will the money buy?

Our pear tree bore eighty-one pears this year. If we eat one-ninth of the pears each day, how many days will they last?

How many square inches will a square nine inches on a side contain? If the square were one inch less on a side, how many square inches would it contain? If it were two inches less on a side, how many square inches would it contain?

In a silk quilt that I am making, some of the squares are three inches on a side and some five inches. How many more square inches in one square than in the other?

How many more squares two inches on a side can you cut from a square six inches on a side than from a square four inches on a side?

How many more squares three inches on a side can you cut from a square nine inches on a side than from one six inches on a side?

The square of nine divided by the square of three are how many? The square of eight divided by the square of four? The square of six divided by the square of two?

The square of nine equals how many nines? The square of seven equals how many sevens? The square of five equals how many fives?

If my watch is thirty seconds slow, what part of a minute is it behind time?

If it is a thirty minutes' walk to school, what part of an hour does it take to walk to school?

What part of an hour are ten minutes? twenty minutes?

Where is the long hand when the time is fifteen minutes past three? ten minutes past three? twenty minutes past three? half past three? a quarter of four? quarter past four?

Where are the hands of a clock at four o'clock? at eight o'clock? at ten o'clock?

What time is it when both hands point to twelve?

What kind of an angle is formed by the two hands of a clock at three o'clock? At what other hour is the same angle formed? What kind of an angle is formed at five o'clock? at eleven o'clock? at one o'clock? What kind of a line is formed by the hands of a clock at six o'clock?

Write in a line the numbers that contain nine without a remainder as far as eighty-one, and see if you can discover anything remarkable about the numbers.

Write in a line the numbers that contain eleven without a remainder as far as seventy-seven, and discover what you can about them.

Write in a line the numbers that contain five without a remainder as far as eighty, and tell me what you can about them.

With what figure does every number end that contains ten without a remainder?

Can you think of any odd number that contains an even number without a remainder?

Can you think of any odd numbers that contain odd numbers without a remainder?

Into how many parts must you divide five to get one for each part? Into how many parts must you divide fourteen to get two for each part? Into how many parts must you divide thirty to get six for each part? forty to get five for each part? fifty-four to get nine for each part? sixty-three to get seven for each part? seventy-two to get six for each part? to get twelve for each part? to get eight for each part? to get nine for each part?

Two pounds are how many ounces?

Two quires of paper are how many sheets of paper?

Five reams of paper are how many quires of paper?

A man buys a stove for old iron, paying \$1.00 for it. He fits it up with a new grate, a new door, and some new

hinges, which cost him \$1.00 all together, and sells the stove for \$7.00. What does he make by his bargain?

How much money will it take for your dictionary at \$2.25, your reader at \$1.25, your geography at \$1.38, your drawing-book at \$0.25, your history at \$0.75, and your pencils and paper at \$0.45? (\$6.33.)

If I buy a barrel of flour for \$7.37, a barrel of sugar for \$12.75, a gallon of molasses for \$0.85, a tub of butter for \$6.68, some meat for \$0.68, and vegetables for \$0.47, what do I pay for the whole? (\$28.80.)

If I read 14 pages in a book one day, 27 another day, 35 another day, 19 another day, 47 another day, 17 another day, and there are 87 more pages in the book, how many pages does the book contain? (246 pages.)

I bought some dress goods for \$17.75, some lace for \$8.37, some buttons for \$0.75, some dress lining for \$0.47, a stick of braid for \$0.06, and some some sewing silk for \$0.28. What was my bill for the whole? (\$27.68.)

Of \$1.87 which I had in my purse I spent \$0.07 for a box of elastic bands, \$0.25 for two quires of paper, \$0.09 for four pencils, and \$0.27 for a book. What had I left? (\$1.19.)

If I earn \$107.00 one month and \$125.00 another month, how much more do I earn one month than the other? (\$18.00.)

If there are 181 children on the upper floor of a room, and 157 on the lower floor, how many more children on the upper floor than on the lower floor? (24 children.)

A bushel of corn weighs 56 pounds. What will 6 bushels weigh? (336 pounds.)

A barrel of flour weighs 196 pounds. What will a load of 8 barrels weigh? (1568 pounds.)

A gross of pens is 144 pens. How many pens in 9 gross? (1196 pens.)

How many sheets of paper in 9 quires of paper? (216 sheets of paper.)

How many ounces in 8 pounds? (128 ounces.)

How many quarts in 8 bushels? (256 quarts.)

In an orchard there were 6 rows of trees with 13 trees in each row, and 37 trees around the outer edge of the orchard. If 58 of these trees died, how many were left? (57 trees.)

A man bought 2 bushels of apples at \$0.75 a bushel, and sold them for \$0.37 a peck. How much did he gain if one peck of apples decayed? (\$1.09.)

If a pound of flour is put into each loaf of bread, how many loaves of bread can be made from 2 barrels of flour? (392 loaves.)

The Rod.

Bertha may take one end of this string and Charles the other end. Stand as far apart as the length of the string. Bertha and Charles are a *rod* apart. How long then is the string they hold?

I have here several other strings just as long. How long is each?

Edna and Mary may take one, and mark off a rod on the platform.

Annie and Harry may take another, and measure off a rod on the long blackboard.

Sarah and Maggie may measure off a rod on the floor.

The others may see if the measuring is correctly done.

How long a distance have you marked off in each case?

John may see how many of his steps it takes to make a rod.

Clarence, place yourself a rod from the door. You are two inches short of a rod.

Clara, place yourself a rod from my table. You are right within half an inch.

What distance in the school yard do you think to be a rod? Who has ever seen any one measure with a rod line?

What is measured by the rod?

I will give each two of you a line which is a rod long, and some time to-day you may measure off a rod in the school yard. Place small sticks to mark the distance.

Yards in a Rod.

Who has found out how many yards there are in the line which measures a rod?

You may measure your rod lines, using the yard lines which I have drawn on the board. How many times do you find the yard to be contained in the rod? Each distance then which you have measured and found to be a rod contains how many yards?

Write: Five and a half yards equal a rod.

Two rods equal how many yards? three rods? seven rods? ten rods? twelve rods?

A rod equals how many feet?

Rods in a Mile.

If I ask how far it is to the next town, will the distance be given me in inches? Will it be given me in feet? in yards? in rods? In what will the distance be given me?

John is right when he says the distance will be given me in miles.

Name a point that is a mile from here; that is two miles; that is several miles.

Name any two points you know that are a mile apart.

What is the longest distance in miles that you have walked at one time?

How many miles do you think all the steps that you take in a day would make? If you should carry in your pocket a little instrument called a pedometer, it would mark the number of steps you take, and, if you know the length of your step, can you find how far you have walked during the day? If, for example, you walk ninety-eight steps, and

your step is two feet, how many feet do you walk? If you take one hundred and fifty steps, how many feet do you walk? If you take eight steps in walking a rod, how many rods do you walk in taking seventy-two steps?

If you should walk *three hundred and twenty* rods, you would walk a *mile*. If you walk two miles, how many rods do you walk? three miles? six miles? seven miles?

If you take eight steps in walking a rod, how many steps will you take in walking a mile?

If you can walk a rod in three seconds of time, how many seconds will it take you to walk a mile?

Write : Three hundred and twenty rods equal a mile.

What is the name of the shortest distance you know? of the next longer distance? of the next longer distance? of the next longer distance? of the longest distance?

Write in a table the number it takes of each of these distances to equal one of the next higher.

You have written the *Table of Long Measure*.

Read the table of long measure.

Roman Numerals.

You may express *one* on the board in all the ways you can. (1, one, I.)

If you look at the clock, you will see another way.

What other letters are used to express numbers on the clock?

There are four other letters used to express numbers : L, C, D, M. How many letters are used to express numbers? Name them. Write them in their order.

A people, called the Romans, first used these letters to express numbers, so we call them the *Roman Numerals*.

Express one with a Roman numeral.

Look at the clock, and then express five with a Roman numeral. Express ten with a Roman numeral.

I will express fifty with a Roman numeral ($L = 50$).

What letter expresses fifty?

What letter expresses one?

What letter expresses five?

What letter expresses ten?

What letter expresses fifty?

What number does I express?

What number does V express?

What number does X express?

What number does L express?

C stands for one hundred.

Express one hundred with a Roman numeral.

Look at the figures I have written on the board. (1, 5, 10, 50, 100.) As I touch each, tell me the letter to write for it?

What letter comes next? (D.)

D stands for five hundred.

Express five hundred with a Roman numeral. ($D = 500$.)

The remaining letter expresses one thousand.

Write: M stands for one thousand. ($M = 1000$.)

Name the letters that are used to express numbers.

What are they called?

Copy and learn: $I = 1$; $V = 5$; $X = 10$; $L = 50$;
 $C = 100$; $D = 500$; $M = 1000$.

Combination of Roman Numerals to form Other Numbers.

What value does I denote?

What value does V denote?

Of these two letters, I and V, place the one denoting the *less* value *before* the other.

Read what you have written (IV).

Subtract the value of the less from the greater, and you will have the number for which IV stands.

This is one way the Romans used to make up their numbers.

What value does X denote?

Of these two letters, I and X, place the one denoting the *less* value *before* the other.

Read what you have written (IX).

Subtract the less from the greater, and you will find what IX stands for.

What value does L denote?

Of these two letters, X and L, place the one denoting the less value before the other.

Read what you have written (XL).

Subtract the value of the less from the greater, and you will find what XL stands for.

What does XC stand for?

As I point to these Roman numerals, you may name the value which they express: I, IV, V, X, L, C, D, M, IV, IX, XL, XC, IX, L, XL, C, XC.

What four new numbers have you made to-day with the Roman numerals? (4, 9, 40, 90.)

What stands for four? What stands for forty?

What stands for nine? What stands for ninety?

When I stands before V, what do you do to find the value of IV?

When I stands before X, what do you do?

When X stands before L, what do you do?

When X stands before C, what do you do?

What Roman numeral stands for one?

Who knows what stands for two? for three?

If two I's stand for two, what should you think two X's would stand for? three X's?

Since two X's stand for twenty, what should you think two C's would stand for? three C's? four C's? two M's? three M's?

Name the number as I show you the Roman numerals:

II, III, IV, V, IX, X, XX, XXX, XL, L, XC, C, CC, CCC, CCCC, D, M, IV, IX, XL, XC, XX, XXX, CC, CCC.

You may write these numbers in Roman numerals : 1, 2, 3, 4, 5, 9, 10, 20, 30, 40, 50, 90, 100, 200, 300, 400.

When two letters of the same value stand together, what do you do to find the value expressed ?

How many ways have you found that the Romans put letters together to make numbers ?

If you look at the clock, you will find how six is made. What forms six ? But where does the less number stand with reference to the greater ? And what do you do to the two numbers to find six ? What other numbers on the clock are made in the same way ?

Who knows any other numbers made in the same way ?

All numbers that you have not already learned to express in Roman numerals are made in this way. ~

See if you can express the numbers between twelve and twenty ; between twenty and thirty ; between thirty and forty ; between forty and fifty.

Add and express the sum in Roman numerals :

Five, seven, six, nine, eight, six.

Twelve, eight, nineteen, three, seven, one.

Three, eight, seven, five, eight, six.

Four, nine, thirteen, three.

Two, seven, six, four.

Seven, eight, ten, twenty-five, eleven, eight.

Eight, nine, seventeen, eight, twenty, nine.

Write in Roman numerals the odd numbers to twelve.

Express all the numbers you can, using but one letter for each number.

Express all the numbers you can, using but two letters for each number.

Express the products of ten as far as one hundred in Roman numerals.

CHAPTER XXXIV.

NUMBERS FROM EIGHTY-FOUR TO ONE HUNDRED.

§ 72. THE NUMBER EIGHTY-FOUR.

Facts to be taught :

12×7	7×12	$\frac{1}{2}$ of 84
$84 \div 7$	$84 \div 12$	$\frac{1}{4}$ of 84
$\frac{1}{12}$ of 84	$\frac{1}{7}$ of 84	2×42

§ 73. THE NUMBER NINETY-SIX.

Facts to be taught :

12×8	8×12	$\frac{1}{3}$ of 96
$96 \div 8$	$96 \div 12$	3×32
$\frac{1}{12}$ of 96	$\frac{1}{8}$ of 96	2×48

Exercise for Review.

Nellie is 9 years old, and Annie is 16. What is the difference of their ages in months?

In a dozen weeks how many days?

If I pay for room rent \$7.00 a month, what is my rent for a year?

A man made a fence across a garden that is 12 yards wide. If he put in posts 2 yards apart to support the railing, how many posts did he use, providing there was a post at each end? How many posts would be required for 12 fences the same length? If the fence formed one straight line, but 12 times as long as the fence across the garden, would 84 posts be required? Why?

A grocer paid \$0.84 for figs at \$0.07 a pound. He lost two pounds by drying and by bad figs. If he sold the rest at \$0.10 a pound, what did he gain on the whole?

How many times does the short hand go round the dial in 84 hours?

While the short hand is going round the dial 7 times, how many times does the long hand go round the dial?

How many times does the short hand go round the dial in a day? How many times will it go round in 42 days?

How broad is a lake which is 42 feet half-way over?

If \$0.12 is a seventh of the price of a book, what is the price of the book?

A strip of ribbon 84 yards in length was cut into 7 equal pieces. How many yards were in each piece? If the ribbon was sold at \$0.84 a piece, what was the price per yard?

If a merchant sells 21 yards of silk for a dress, how many yards are in a piece of goods from which he can sell 4 dresses?

What four equal numbers make 84? What, then, is one-fourth of 84?

If there are 84 windows in a building, and one-fourth of this number is in the front of the building, how many windows are on the two sides and at the back?

If \$0.84 will buy 12 car-tickets, what is the price of one car-ticket?

A man charged \$0.84 a yard for carpeting, but at length agreed to deduct one-twelfth of that price. What, then, was his price for the carpeting? How much less did 12 yards cost than if the first price had been paid? How many yards of carpet lining would that buy at \$0.07 a yard?

If a peck of berries cost \$0.96, what is that a quart?

Into how many piles can I separate 96 oranges, if I put a dozen in each pile?

A man has 4 baskets, each containing 24 apples. If he sells them at \$0.05 a dozen, what will he receive?

A man had 96 books. He sold 84 books. What part of the whole number had he left?

A man shipped eight dozen boxes of oranges from Florida. When the oranges arrived, $\frac{1}{12}$ was found to have decayed. How many boxes were good?

A train travels ninety-six miles in three hours. What is the rate of speed per hour?

A man bought a bushel of peaches at \$0.03 a quart. What did he pay for the bushel of peaches?

If a large cask holds 48 gallons, what will 2 casks hold?

A man works 96 hours in a week and two days. How many hours a day does he work?

If a pencil costs \$0.05 and an eraser costs \$0.03, what will a dozen of each cost together? How many cents less than a dollar?

At \$0.08 a card, how many cards can I buy for \$0.96?

Charles gets \$6.00 a month for helping his father. Emma gets $\frac{1}{4}$ as much for helping her mother. How much will both have earned in a year?

How broad is a house which is 3 rooms in breadth, each room being 15 feet broad, the two outer walls being each 2 feet thick, and the partitions between the rooms each $\frac{1}{2}$ a foot thick?

How many rods are there in 3 miles?

How many hours are there in a week?

If an orange is worth 3 apples, how many apples are a dozen oranges worth? How many oranges are a dozen apples worth?

How far is it around a square which is 19 feet on a side?

If one scale pan had 25 pounds in it, and the other had 13 pounds in it, how many pounds must be taken from one and put into the other to make the scales balance?

Into how many bunches, containing 5 bananas each, can you separate a bunch of 40 bananas? a bunch of 60 bananas? a bunch of 80 bananas?

If you use a drop of ink at 2 dips, how many dips can you take from a bottle containing 125 drops, if 15 drops dry up?

If I packed 84 pairs of shoes in 4 boxes, putting an equal number of pairs in each box, how many pairs did I put in each box?

A man had a dozen boxes of candy, and 8 pounds in each box. If he put this candy up in packages containing a quarter of a pound each, how many packages will he have?

If while you were pumping 20 gallons of water out of a well, one gallon ran in, how many gallons less were there in the well after you pumped the water than before?

If each one of a family of 8 persons use 2 gallons of water in a day, how much will they use in a week?

A quire of paper will make how many strips of paper, if each sheet is cut into four strips?

If I start from my home and travel 4 miles in one direction, and then turn about and travel 5 miles from my home in the opposite direction, how many miles do I travel before I reach home?

If I go to West Rock, which is 3 miles from my home, and then to East Rock, which is 5 miles from my home in the opposite direction, how far do I travel before I reach home?

A river at low water was 200 feet wide, but at high water it spread over the shore on one side 25 feet, and on the other side 38 feet. How wide was the river at high water?

What will 4 dozen plates, at \$1.87 a dozen, and 6 dozen saucers, at \$0.98 a dozen, cost?

Which is the cheaper, boots at \$3.00 a pair, if I wear out 4 pairs in a year, or boots at \$8.00 a pair, if I wear out 3 pairs in two years?

If you have a half a dollar, how many quarters of a dollar is it worth? how many eighths of a dollar?

If you have a half of a quire of paper, how many sixths of a quire have you? how many twelfths of a quire?

If Jennie is $\frac{1}{2}$ a score of years old, and her brother is $\frac{1}{10}$ of a score of years old, how many times as old as her brother is Jennie?

If it takes $\frac{1}{3}$ of a yard of ribbon for a loop, how many loops will $\frac{1}{3}$ of a yard of ribbon make?

It is required to cut a loaf of cake into ninths. It is already cut into thirds. Into how many pieces must I cut each third to cut the whole into ninths?

If you sleep $\frac{1}{3}$ of the day, how many twelfths of the day do you sleep?

$\frac{1}{2}$ equals how many fourths? equals how many sixths? equals how many eighths? equals how many tenths? equals how many twelfths?

$\frac{1}{3}$ equals how many sixths? equals how many ninths? equals how many twelfths?

$\frac{1}{4}$ equals how many eighths? equals how many twelfths?

$\frac{1}{5}$ equals how many tenths?

$\frac{1}{6}$ equals how many twelfths?

$\frac{2}{3}$ equal how many ones?

$\frac{3}{4}$ equal how many ones?

$\frac{4}{5}$ equal how many ones?

How many fifths in a whole?

How many sixths in a whole?

How many sevenths in a whole?

In 3 oranges how many halves?

In 4 dollars how many fourths?

In 2 yards of ribbon how many sixths?

In 5 pounds how many thirds of a pound?

In 2 dollars how many fifths of a dollar?

In 3 yards how many sevenths of a yard?

In 4 bushels how many eighths of a bushel?

If you have $\frac{2}{3}$ of a dollar and I have $\frac{1}{3}$, how many fifths have we together?

If a grocer has $\frac{3}{8}$ of a bushel of beans of one kind, and $\frac{4}{8}$ of a bushel of another kind, how many eighths has he of both kinds?

If a salesman sells $\frac{1}{3}$ of a yard of ribbon to one person, $\frac{1}{3}$ to another, and $\frac{1}{3}$ to another, how many sixths does he sell all together?

If there is $\frac{1}{2}$ of a yard of ribbon in one piece and $\frac{1}{2}$ of a yard in another piece, how many fourths of a yard are in the two pieces together?

If I buy $\frac{1}{2}$ a pound of one kind of dates and $\frac{1}{2}$ of a pound of another kind, how many eighths of a pound do I buy in all?

If I have $\frac{1}{2}$ of a yard of velvet and $\frac{1}{2}$ of a yard, how many sixths of a yard do I have?

If it is $\frac{1}{2}$ of a mile to the post-office and $\frac{1}{10}$ of a mile farther to the depot, how many tenths of a mile is it to the depot?

I have $\frac{1}{2}$ of a quire of letter paper and $\frac{1}{12}$ of a quire of note paper. How many twelfths of a quire of paper have I all together?

If John gathers $\frac{1}{4}$ of a bushel of walnuts and $\frac{1}{8}$ of a bushel of chestnuts, how many eighths of a bushel does he gather of both kinds?

If a small corner of our yard is $\frac{1}{4}$ of a rod on one side and $\frac{1}{12}$ of a rod on the other side, how many twelfths of a rod is it on both sides?

If it is $\frac{1}{3}$ of a mile to Walnut Hill and $\frac{1}{6}$ of a mile up the hill, how many sixths of a mile is it from here to the top of the hill?

If the dado of a room is $\frac{1}{3}$ of the height of the room, and the border at the top of the room is $\frac{1}{3}$ of its height, how many ninths of the height of the room are both dado and border?

If I find $\frac{1}{3}$ of a dozen of eggs in one nest and $\frac{1}{12}$ of a dozen in another nest, how many twelfths of a dozen of eggs do I find in both nests?

If you have $\frac{1}{3}$ of a dollar and I have $\frac{1}{10}$ of a dollar, how many tenths have we together?

If you are in school $\frac{1}{3}$ of the day and study at home $\frac{1}{12}$ of the day, how many twelfths of the day do you give to school work?

Fred had 50 dollars, and spent $\frac{1}{2}$ of it for a watch, and $\frac{1}{10}$ of it for a pair of shoes. How many tenths of it did he spend?

A grocer sold $\frac{1}{4}$ of a bushel of pears to one customer, $\frac{1}{8}$ to another, and $\frac{3}{16}$ to another. How many sixteenths of a bushel did he sell?

John spent $\frac{2}{3}$ of a dollar for a book, $\frac{1}{10}$ of a dollar for half a ream of paper, and had $\frac{1}{2}$ of a dollar left. How much money had he at first?

James paid $2\frac{1}{2}$ dollars for a hat, $3\frac{1}{2}$ dollars for a vest, and $10\frac{1}{2}$ dollars for a coat. What did the whole cost?

Edgar can do a certain piece of work in 3 days, and George can do the same work in 6 days. What part of the work can Edgar do in 1 day? What part of the work can George do in 1 day. What part of the work can both together do in 1 day?

A grocer sold $\frac{1}{3}$ of a dozen of oranges to one man, $\frac{1}{3}$ of a dozen to another, and $\frac{5}{6}$ of a dozen to another, and had 2 dozen left. How many oranges had he at first?

How many yards is it half-way round a flower-bed which measures 18 feet on each side, and 12 feet on each end?

CHAPTER XXXV.

PROCESSES IN WRITTEN ARITHMETIC CONTINUED.

§ 74. THIRD STEP IN MULTIPLICATION.

When the multiplier consists of two figures and the left hand figure is 1.

I have 14 boxes with three dozen eggs in each box. I want you to find how many eggs I have. Express the example on the board. You may first find how many eggs I have in 4 boxes. How many more boxes have I? Find then how many eggs I have in 10 boxes.

How many eggs in 4 boxes? (144.)

How many eggs in 10 boxes? (360.)

How many eggs in 14 boxes?

Then 14 times 36 are how many?

Frank earned \$0.75 a day. How much did he earn in 13 days? Find first how much he earned in 3 days; then how much he earned in 10 days; and at last how much he earned in the whole time.

In 16 bushels how many quarts? First find how many quarts in 6 bushels; next, how many quarts in 10 bushels, and then how many quarts in 16 bushels.

If a bushel of corn meal weighs 56 pounds, how many pounds will a bin which holds 15 bushels contain? What shall we do first? (Find how many pounds in a bin which holds 5 bushels.) What next? (Find how many pounds in a bin which holds 10 bushels.) What next? (Add, to find how many pounds in 15 bushels.)

An engineer rides 97 miles in a day. How many miles does he ride in three weeks if his train does not run Sundays? What shall we do first? (Find how far he rides in 8 days.) What next? (Find how far he rides in 10 days.) What last? (Add, to find how far he rides in 18 days.)

A merchant sold in one month 17 silk cloaks at \$17 each. What did he receive for all of them together?

If I have a square that is 18 feet on a side, how many square feet are there in the square?

NOTE.—In a short time the child discovers that he can multiply by units of tens as easily as by single units, always remembering that the first product is tens. When he arrives at this knowledge he can multiply by any number of tens, and is ready to pass to the next step in multiplication.

Further steps in Multiplication.

36	364	225	367
47	47	247	105
—	—	—	—
360	64	64	64
105	10	100	1000
—	—	—	—
64	64	64	6400
30	300	3000	3000
—	—	—	—

Multiplications that can be Performed by Short Methods.

1. Multiplication by 9.

Annex a cipher, and subtract the number to be multiplied.

2. Multiplication by 11.

Place the sum of the digits between the digits.

3. Multiplication by 25.

Annex two ciphers, and divide by four.

4. Finding the square of a number whose units' figure is 5.

Multiply the tens by the next higher number of tens, and add the square of 5.

5. Multiplication of two numbers whose units are five and the difference of whose tens is 1.

Square the highest number of tens, and from the result subtract the square of 5.

6. Multiplication of two numbers the sum of whose units is ten, and the difference of whose tens is one.

Square the highest number of tens, and from the product subtract the square of the units of the highest number.

§ 75. FIRST STEP IN DIVISION.

When the dividend is a multiple of ten, and the number of tens is a multiple of the divisor, and the divisor is a number of units.

Which is more, two tens or ten twos?

Which is more, six tens or ten sixes?

Which is more, four tens or ten fours?

Which is more, seven tens or ten sevens?

Which is more, nine tens or ten nines?

Two tens equal how many *twos*?

Three tens equal how many threes?

Four tens equal how many fours?

Six tens equal how many sixes?

Eight tens equal how many eights?

Nine tens equal how many nines?

Each take this number of splints. (60.) How many *two-tens* have you? Since two tens and ten twos are the same, *three* two-tens are how many ten-twos? (Three ten-twos.)

How many three-tens do you find in 60? Then how many ten-threes? (Two ten-threes.)

How many six-tens do you find?

How many ten-sixes then?

Take 120 splints. How many two-tens in 120? How many ten-twos?

How many three-tens in 120? How many ten-threes?

How many six-tens in 120? How many ten-sixes?

Take 90 splints. How many three-tens in 90? How many ten-threes?

Take 140 splints. How many two-tens in 140? How many ten-twos? How many seven-tens in 140? How many ten-sevens?

How many ten-twos in 80? * How many ten-fours in 80?

How many ten-twos in 100? How many ten-fives in 100?

How many ten-twos in 160? How many ten-fours in 160? How many ten-eights in 160?

How many ten-twos in 40? *Two* (times) *ten*-twos are how many *twos*? Then how many *twos* in 40?

How many ten-twos in 60?

Three (times) ten-twos are how many twos? Then how many *twos* in 60?

See if you can find how many twos in 80. Find the *ten*-twos first, and then you can tell how many *twos*.

Find how many twos in 100. How many twos in 120? How many twos in 140?

Find how many *threes* in 60. First find how many *ten*-threes, and you can tell how many *threes*.

Find the number of threes in 90; in 120; in 150.

Find the number of fours in 80; in 120; in 160.

Find the number of fives in 100; in 150.

Find the number of sixes in 120; in 180.

* Use splints.

Find the number of sevens in 140.

Find the number of eights in 160.

Find the number of nines in 180.

Express on the board, forty divided by two; eighty divided by two; one hundred divided by two; one hundred and twenty divided by two; one hundred and forty divided by two.

Express, sixty divided by three; ninety divided by three; one hundred and twenty divided by three; one hundred and fifty divided by three; eighty divided by four; one hundred and twenty divided by four; one hundred and fifty divided by five; one hundred and eighty divided by six; one hundred and forty divided by seven; one hundred and sixty divided by eight; one hundred and eighty divided by nine.

§ 76. SECOND STEP IN DIVISION.

When each order in the dividend is exactly divisible by the divisor.

You may express the division of forty-four by two.

How many twos in forty?

How many twos in four?

Then how many twos in forty-four?

Express the division of sixty-nine by three.

How many threes in sixty?

How many threes in nine?

Then how many threes in sixty-nine?

Express the division of eighty-eight by four.

How many fours in eighty?

How many fours in eight?

Then how many fours in eighty-eight?

Divide sixty-four by two.

Divide forty-four by two.

- Divide sixty-six by three.
- Divide ninety-three by three.
- Divide ninety-six by three.
- Divide ninety-nine by three.
- Divide one hundred and twenty-six by three.
- Divide one hundred and twenty-nine by three; fifty-five by five; one hundred and five by five; one hundred and twenty-six by six.

Further steps in division :

$$6 \overline{)96}$$

$$3 \overline{)37}$$

$$4 \overline{)136}$$

$$8 \overline{)240}$$

$$2 \overline{)402}$$

$$7 \overline{)721}$$

$$13 \overline{)57(}$$

$$13 \overline{)152(}$$

$$13 \overline{)141(}$$

Exercise for Review.

A boy picked from one vine enough grapes to fill a bushel measure, a half-bushel measure, and a peck measure. How many pecks of grapes did he pick? If he sells the grapes at \$0.07 a quart, what does he get for a peck? If he had sold the grapes for \$0.06 less per peck, what would he have received for the whole lot? By selling his grapes at \$0.50 a peck, how much less money would he get for the whole than by selling them at \$0.07 a quart?

How much oil do we use in 8 weeks if we use a quart each evening?

If you put a bushel of corn, which weighs 56 pounds, into 8 baskets, each holding the same amount, how many pounds of corn will each basket contain?

A man puts 56 quarts of milk into cans holding 8 quarts each, but does not fill the cans within a quart each. How many more cans will he use than if he filled each can?

A man who carried 56 eggs to market found $\frac{1}{4}$ of the number broken when he unpacked them. If eggs were selling for \$0.03 each, how much did he lose?

If you earn \$56.00 in 7 weeks, what is that per week?

If Robert found 9 hens' nests with a half dozen eggs in each nest, and left one egg in each nest for the nest-egg, how many eggs did he take away?

How many square inches in a square 7 inches on a side?

How many inches wide is the window of a conservatory for flowers if there are 9 panes of glass across it, each a half foot in length, and the bars take up six inches?

In 9 pecks of beans how many quarts of beans, if they shrink in drying one quart to a peck?

A man gave his note payable in 60 days. How many weeks was the note to run if he was allowed 3 more days?

A stationer bought 12 dozen pens at \$0.05 a dozen, and sold them two for a cent. What did he gain?

In $\frac{1}{4}$ of an hour how many minutes?

If the horse-cars going the same way pass my house five times an hour, how often do they run? If they pass twelve times an hour, how often do they run?

If in going in one direction the cars pass every half hour, how often do they pass in going both directions?

If it takes a car twenty minutes to make a trip, and the cars pass each other half way on the trip, how often do the cars pass each other? If the two cars start at opposite ends of the route at the hour, at twenty minutes past the hour, and at twenty minutes of the hour, at what times through the day do they meet?

If a trip in the cars is 9 minutes long, how many minutes does it take the cars to make 8 trips? How many minutes over an hour?

If it is 9 minutes' ride in the cars out to Lakewood, how many minutes does it take a car to go four times? five times?

If the cook fries 72 oysters, thus allowing 9 oysters to each person at the table, how many persons are expected at table? If each person ate only 8 oysters, how many would be left for the cook?

If I have 72 pounds of flour, and use $\frac{1}{2}$ of it at each baking, for how many bakings will it be sufficient? How many pounds shall I use at each baking?

How many square feet in a square that is three feet on a side? If the square is three feet on a side, how many yards on a side is it? Then in a square yard how many square feet? In 9 square yards how many square feet?

In a square that is 9 yards on a side how many square yards? Which do you think is the larger area, 9 square yards or a square 9 yards on a side?

A pole 7 feet long is how many inches long?

A man 6 feet tall measures how many inches in height?

If 8 benches, each a foot from the floor, are piled top of each other, how much higher will they be than I, who am 5 feet and 6 inches? If I can reach up two feet above my height, how near can I come to reaching the top bench?

If a boy earns \$8.00 per month, what will he earn in a year? If he spends one-third of this sum for clothes, how much has he left?

In one gallon how many gills?

In one bushel how many pints?

In one rod how many yards?

In one rod how many feet?

In one rod how many inches?

In one week how many hours?

In half a day how many minutes?

In an hour how many seconds?

If 12 dozen pens are a gross of pens, how many pens in a gross?

In 16 pounds how many ounces?

A furlong is an eighth of a mile. How many rods are in a furlong?

It is $\frac{5}{8}$ of a mile to school. How many eighths less than a mile is it?

If you have $\frac{3}{4}$ of a dollar and spend $\frac{1}{4}$, how many fourths of a dollar have you left?

I ordered a pound of cheese, but the grocer cut off only $\frac{3}{4}$ of a pound. How much less than a pound did I get?

If the milliner uses $\frac{5}{8}$ of a yard of one kind of velvet in trimming a hat and $\frac{3}{8}$ of a yard of another kind of velvet, how much more of one kind does she use than of the other?

If I have $\frac{1}{2}$ of a dollar and you have $\frac{1}{4}$ of a dollar, how many more fourths have I than you?

If I buy $\frac{1}{2}$ of a pound of sugar and $\frac{1}{4}$ of a pound of tea, how much more sugar than tea do I buy?

Of $\frac{1}{2}$ a pound of meat, $\frac{1}{4}$ of a pound was fat and gristle. How many sixths of a pound was lean?

I had $\frac{1}{2}$ of a yard of smilax, but $\frac{1}{10}$ of a yard got broken off. How many tenths of a yard had I then?

I had a half-foot ruler, but my little brother whittled off of one end $\frac{1}{12}$ of a foot. How many twelfths of a foot long was my ruler then?

I had $\frac{1}{4}$ of a quire of paper, but have used $\frac{1}{8}$ of a quire of the paper. How many eighths of a quire have I now?

If you have $\frac{1}{4}$ of a ton of coal in a bin, and use $\frac{1}{12}$ of a ton, how many twelfths of a ton remain in the bin?

If you have $\frac{1}{3}$ of a bunch of envelopes, and use $\frac{1}{6}$ of a bunch, how many sixths of a bunch remain?

A basket and some grapes in the basket together weigh $\frac{1}{2}$ of a pound. If the basket weigh $\frac{1}{4}$ of a pound, how many ninths of a pound do the grapes weigh?

A boy, in carrying a measure which contained $\frac{1}{2}$ of a bushel of meal, spilt $\frac{1}{12}$ of a bushel. How many twelfths of a bushel were then in the measure?

It is $\frac{1}{4}$ of a mile across the green. $\frac{1}{12}$ of a mile is shaded. What part of a mile is not shaded?

If one furlong is $\frac{1}{8}$ of a mile, how many eighths of a mile are five furlongs?

If it takes $\frac{2}{3}$ of a yard of lace for one sleeve, how many sixths will it take for 2 sleeves? for 3? for 4? for 6?

If it is $\frac{3}{4}$ of a mile to town, and I walk the distance four times a day, how many thirds of a mile do I walk? how many miles?

If a man is 28 years old and his son is $\frac{1}{4}$ as old, how old is the son?

If you have collected four dozen stamps, of which one-fourth are foreign stamps, how many foreign stamps in your collection?

If a yard of cloth costs \$1.37, what will 7 yards cost?

If there were but 30 days in each month, how many days would there be in a year?

If I have 25 quarters of a dollar, what number of cents do I have?

I bought 19 yards of silk at \$0.75 a yard, and 3 yards of velvet at \$1.50 a yard. What was the amount of my bill?

What will 95 street lamps cost at \$11.00 each?

If a cannon-ball weighs 12 pounds, what will a dozen cannon-balls weigh?

If a bale of hay weighs 280 pounds, what will 65 bales weigh?

If your pulse beats 65 times a minute, how many times will it beat in an hour?

If from the sea to the base of a hill there is a rise of 425 feet, and the hill is 2 times as high as this rise, how far above the sea is the summit of the hill?

It is 160 rods across a pond, and twice as far around the pond; how many rods do I row if I cross the pond, go entirely round the pond, and then cross again to the place from which I started?

How many sheets of paper in a ream of paper?

In 100 pints how many quarts?

In 80 gills how many pints?

In 160 quarts how many gallons?

In 240 quarts how many pecks?

In 99 feet how many yards?

In a mile how many spaces, each 8 rods long, can you lay off?

If a man has a piece of ground containing 180 square feet, and it takes 9 square feet to make a square yard, how many square yards in his piece of ground?

If in a bed of plants there are 155 plants set out in rows with 5 plants in each row, how many rows of plants in the bed?

If a cord of wood is worth \$6, and 10 cords of wood are given for 12 thousand shingles, how much are the shingles a thousand?

A man bought an equal number of pigs and sheep for \$63. Each pig cost \$3, and each sheep \$4. How many of each did he buy?

A woman bought 24 yards of gingham at \$0.30 a yard, and 23 yards of cotton cloth at \$0.10 a yard. How much change did she receive out of a ten-dollar bill?

A man can do a piece of work in 4 days by working 12 hours a day. How many days will be required if he works only 8 hours a day?

A man can do a piece of work in 6 days by working 8 hours a day. How many hours a day must he work to finish it in 4 days?

Two men are 96 miles apart, and are travelling toward each other, one at the rate of 5 miles an hour, and the other at 3 miles an hour. In how many hours will they meet?

If 2 yards of silk cost \$4, what will 5 yards cost?

CHAPTER XXXVI.

FRACTIONS.

§ 77. REDUCTION OF FRACTIONS.

First Step. Reduce improper fractions to whole or mixed numbers. Thus, $\frac{2}{2} = 1$; $\frac{4}{4} = 1$; $\frac{8}{8} = 1$; $\frac{9}{3} = 3$; $\frac{7}{4} = 1\frac{3}{4}$.

Second Step. Reduce to equivalent fractions of higher terms. Thus, $\frac{1}{2} = \frac{2}{4}$; $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{4} = \frac{2}{8}$; $\frac{1}{8} = \frac{2}{16}$.

Third Step. Reduce to equivalent fractions of lower terms. Thus, $\frac{2}{4} = \frac{1}{2}$; $\frac{3}{6} = \frac{1}{2}$; $\frac{2}{8} = \frac{1}{4}$; $\frac{2}{16} = \frac{1}{8}$.

Fourth Step. Reduce a whole or mixed number to an improper fraction. Thus, $2 = \frac{4}{2}$; $2 = \frac{8}{4}$; $2\frac{1}{2} = \frac{5}{2}$; $2\frac{3}{4} = \frac{11}{4}$.

Fifth Step. Reduce to least common denominator.

Exercise for Fourth Step.

If I get a two-dollar bill changed, receiving half-dollars for it, how many half-dollars shall I receive?

If you lend me 2 dollars, and I return the money in quarters, how many quarters must I give you?

If you cut 2 pies of equal size into eighths, how many eighths of a pie will you make?

If you cut two saucer pies into thirds, how many thirds of a pie will you make?

If you cut 2 yards of cloth into strips, each $\frac{1}{8}$ of a yard wide, how many strips will you have?

If a man puts two gallons of cider into bottles, each holding $\frac{1}{2}$ of a gallon, how many bottles will he use?

If we burn $\frac{1}{4}$ of a ton of coal a day, how long will 2 tons last us?

Express this number (2) in halves; in fourths; in eighths; in thirds; in sixths; in sevenths; in fifths; in ninths; in twelfths.

Express this number (4) in halves; in thirds; in fourths; in eighths.

Express any number you please in halves. Express the same number in other fractions.

If it is $2\frac{1}{2}$ miles to West Rock, how many halves of a mile is it to West Rock?

If I buy $2\frac{1}{3}$ yards of ribbon, how many thirds of a yard do I buy?

If you have a 2-dollar bill and a quarter of a dollar, and I have the same amount of money in quarters, how many quarters have I?

If I have $2\frac{1}{5}$ barrels of kindling wood, how many fifths of a barrel do I have?

If a strip of carpeting is $2\frac{1}{6}$ yards long, how many sixths of a yard in the strip?

Express $2\frac{1}{7}$ in sevenths.

Express $2\frac{1}{8}$ in eighths.

Express $2\frac{1}{9}$ in ninths.

Express $2\frac{1}{12}$ in twelfths.

Express the numbers I name, in halves: $4\frac{1}{2}$, $7\frac{1}{2}$, $10\frac{1}{2}$, $11\frac{1}{2}$.

Express in fourths: $3\frac{1}{4}$, $3\frac{3}{4}$, $5\frac{1}{4}$, $6\frac{3}{4}$, $9\frac{1}{4}$, $11\frac{3}{4}$.

Express in thirds: $5\frac{1}{3}$, $5\frac{2}{3}$, $7\frac{1}{3}$, $7\frac{2}{3}$, $12\frac{1}{3}$, $11\frac{2}{3}$, $9\frac{2}{3}$.

Express in eighths: $2\frac{3}{8}$, $2\frac{5}{8}$, $5\frac{1}{8}$, $7\frac{7}{8}$, $9\frac{3}{8}$, $11\frac{5}{8}$, $12\frac{7}{8}$.

Express in sixths: $2\frac{2}{3}$, $3\frac{2}{3}$, $4\frac{5}{6}$, $6\frac{1}{6}$.

Express in fifths: $2\frac{3}{5}$, $3\frac{2}{5}$, $8\frac{3}{5}$, $9\frac{4}{5}$.

Express in sevenths: $2\frac{4}{7}$, $3\frac{3}{7}$, $4\frac{6}{7}$, $5\frac{5}{7}$, $6\frac{3}{7}$.

Exercise for Fifth Step.

I bought $\frac{1}{2}$ of a yard of one kind of satin and $\frac{1}{3}$ of a

yard of another kind of satin. Who can tell me in sixths what part of a yard of each kind I bought?

If I pay $\frac{1}{2}$ of a dollar for a book and $\frac{1}{4}$ of a dollar for some note paper, how many tenths of a dollar do I pay for each?

If I have $\frac{1}{2}$ of a cord of hard wood and $\frac{1}{4}$ of a cord of kindling wood, how many fourteenths of a cord of each kind do I have?

If you are in one school $\frac{1}{2}$ of the year and in another $\frac{1}{3}$ of a year, how many eighteenths of a year are you in each school?

Who will express on the board $\frac{1}{2}$ and $\frac{1}{3}$ in fractions having their denominators alike?

Who will express $\frac{1}{3}$ and $\frac{1}{4}$ in fractions having their denominators alike? $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{1}{2}$ and $\frac{1}{5}$?

Who will express $\frac{1}{5}$ and $\frac{1}{4}$ in fractions having their denominators alike? $\frac{1}{5}$ and $\frac{1}{5}$? $\frac{1}{4}$ and $\frac{1}{5}$? $\frac{1}{4}$ and $\frac{1}{6}$? $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$? $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$? $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{10}$? $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{12}$? $\frac{1}{2}$, $\frac{1}{7}$, and $\frac{1}{14}$? $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$? $\frac{1}{5}$ and $\frac{1}{6}$? $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{6}$? $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{12}$? $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{15}$?

When you change fractions to other fractions whose denominators are alike, we say that you *reduce the fractions to a common denominator*. When you changed $\frac{1}{2}$ and $\frac{1}{3}$ to $\frac{2}{6}$ and $\frac{2}{6}$, you reduced $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator. When you changed $\frac{1}{2}$ and $\frac{1}{3}$ to tenths, what do we say you did? When you changed $\frac{1}{3}$ and $\frac{1}{4}$ to twelfths, what did you do? When I ask you to reduce $\frac{1}{2}$ and $\frac{1}{4}$ to a common denominator, to what will you change the fractions? When you reduce $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$ to a common denominator, to what will you change the fractions?

Reduce to a common denominator $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Reduce to a common denominator $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$.

Reduce to a common denominator $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{2}{3}$ and $\frac{1}{4}$; $\frac{2}{3}$, $\frac{2}{6}$, and $\frac{1}{12}$; $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{2}{9}$.

§ 78. ADDITION OF FRACTIONS.

First Step. Add fractions whose denominators are alike and whose sum is less than one. Thus, $\frac{2}{8} + \frac{2}{8}$; $\frac{1}{3} + \frac{1}{3}$; $\frac{2}{4} + \frac{1}{4}$; $\frac{1}{5}, \frac{2}{5}, \frac{1}{5}$; $\frac{1}{6}, \frac{2}{6}, \frac{2}{6}$.

Second Step. Add fractions whose denominators are unlike, but whose least common denominator is found in one of the fractions to be added. Thus, $\frac{1}{2} + \frac{1}{4}$; $\frac{1}{2} + \frac{1}{8}$; $\frac{1}{4} + \frac{1}{8}$; $\frac{1}{3} + \frac{1}{6}$; $\frac{1}{2} + \frac{1}{6}$; $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$; $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$; $\frac{2}{3} + \frac{2}{3}$.

Third Step. Add fractions whose denominators are unlike and whose least common denominator is not found in any one of the fractions to be added. Thus, $\frac{1}{2} + \frac{1}{3}$; $\frac{1}{3} + \frac{1}{5}$.

Fourth Step. Add mixed numbers. Thus, $3\frac{1}{2} + 5\frac{1}{4}$.

Exercise for Third Step.

It takes $\frac{1}{2}$ of a yard of lace for my sleeves, and $\frac{1}{3}$ of a yard for my collar. I want you to find what part of a yard it takes for both neck and sleeves. Who can tell me the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$? Then give me your answer in sixths.

A man earns $\frac{1}{2}$ of a dollar an hour, and his son earns $\frac{1}{3}$ of a dollar an hour. What part of a dollar do they earn together? What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$? Then find your answer in tenths.

It is $\frac{1}{3}$ of a mile to the post-office and $\frac{1}{4}$ of a mile farther to the depot. What part of a mile is the distance to the depot? To what will you reduce the fractions? What is your number of twelfths?

Find the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. What is the least common denominator?

Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$.

Find the sum of $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{6}$.

Find the sum of $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{6}$.

Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

Find the sum of $\frac{1}{3}$ and $\frac{2}{4}$; $\frac{1}{4}$ and $\frac{2}{3}$.

Find the sum of $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$; $\frac{5}{8}$ and $\frac{2}{3}$.

Exercise for Fourth Step.

In one piece of trimming there are $4\frac{1}{2}$ yards, and in another piece of the same trimming there are $11\frac{1}{3}$ yards. I want you to find how many yards there are in both pieces. You may add your fractions *first* in all such numbers. What is the sum of the fractions? What is the sum of the other numbers? What is the entire sum?

Try if you can add $19\frac{1}{3}$ and $17\frac{1}{4}$; $15\frac{1}{2}$ and $75\frac{1}{3}$; $12\frac{1}{4}$ and $9\frac{1}{3}$; $8\frac{2}{3}$ and $7\frac{1}{3}$; $9\frac{1}{4}$ and $6\frac{2}{3}$; $5\frac{1}{2}$ and $9\frac{3}{4}$.

§ 79. SUBTRACTION OF FRACTIONS.

First Step. Subtract fractions having like denominators. Thus, $\frac{2}{4} - \frac{1}{4}$; $\frac{5}{8} - \frac{2}{8}$; $\frac{4}{7} - \frac{2}{7}$; $\frac{5}{9} - \frac{2}{9}$.

Second Step. Subtract fractions whose denominators are unlike, but whose least common denominator is found in one of the given fractions. Thus, $\frac{1}{2} - \frac{1}{4}$; $\frac{2}{3} - \frac{1}{6}$; $\frac{3}{4} - \frac{1}{2}$.

Third Step. Subtract fractions whose denominators are unlike, but whose least common denominator is not found in any one of the given fractions. Thus, $\frac{1}{2} - \frac{2}{3}$; $\frac{1}{2} - \frac{3}{4}$.

Fourth Step. Subtract one mixed number from another.

Exercise for Third Step.

From $\frac{1}{2}$ of a barrel of vinegar I have drawn off $\frac{1}{3}$ of a barrel of vinegar. What part of a barrel of vinegar remains?

What must you do before you can find the difference between $\frac{1}{2}$ and $\frac{1}{3}$? What is the least common denominator? What do you find your answer to be?

A book which I bought cost $\frac{1}{3}$ of a dollar. I paid for it with $\frac{1}{2}$ of a dollar. What part of a dollar ought I to receive in change?

Find the difference between $\frac{2}{3}$ and $\frac{1}{2}$.

Find the difference between $\frac{3}{4}$ and $\frac{1}{2}$.

Find the difference between $\frac{4}{5}$ and $\frac{1}{2}$.

Find the difference between $\frac{5}{6}$ and $\frac{1}{2}$; $\frac{2}{3}$ and $\frac{1}{3}$.

Find the difference between $\frac{2}{3}$ and $\frac{2}{5}$; $\frac{4}{5}$ and $\frac{3}{4}$.

Exercise for Fourth Step.

There were $2\frac{1}{2}$ yards cut from a pole which was a rod in length. I wish to know how many yards in length the pole was then. If only $\frac{1}{2}$ of a yard had been taken from $5\frac{1}{2}$ yards, how long would the pole be? If 2 yards more were cut off, what then remained? How long then was the pole that lacked $2\frac{1}{2}$ yards of being a rod long?

If a room is $10\frac{1}{2}$ feet high, and the bordering and dado take up $4\frac{1}{2}$ feet, what is the distance between the bordering and the dado? Find first what would be the distance if only $\frac{1}{2}$ foot were taken up. Find next what was the distance when 4 more feet were taken.

Find the difference between $8\frac{2}{3}$ and $3\frac{1}{3}$; $72\frac{3}{4}$ and $19\frac{1}{4}$; $70\frac{7}{8}$ and $15\frac{1}{4}$; $63\frac{3}{8}$ and $29\frac{1}{8}$; $85\frac{5}{8}$ and $27\frac{3}{4}$; $62\frac{5}{8}$ and $37\frac{1}{4}$.

§ 80. MULTIPLICATION OF FRACTIONS.

First Step. Multiply a fraction by an integer. Thus, 2 times $\frac{2}{3}$; 3 times $\frac{2}{3}$; 3 times $\frac{5}{7}$.

Second Step. Find fractional parts of numbers. Thus, $\frac{1}{2}$ of 16; $\frac{2}{3}$ of 12; $\frac{3}{4}$ of 20; $\frac{3}{4}$ of 24; $\frac{5}{6}$ of 30; $\frac{4}{7}$ of 28.

Third Step. Multiply one fraction by another when the operation can be performed by dividing the numerator of the multiplicand by the denominator of the multiplier. Thus, $\frac{1}{2}$ of $\frac{4}{7}$; $\frac{1}{2}$ of $\frac{6}{7}$; $\frac{1}{3}$ of $\frac{9}{11}$; $\frac{1}{4}$ of $\frac{8}{9}$; $\frac{1}{5}$ of $\frac{15}{8}$.

I had $\frac{8}{9}$ of a dollar, and spent $\frac{1}{2}$ of what I had. Let us find how many eighths of a dollar I spent. If the eighths of a dollar had been whole dollars, and I spent $\frac{1}{2}$ of it, how many dollars would I have spent? If $\frac{1}{2}$ of 6 dollars is 3 dollars, then $\frac{1}{2}$ of $\frac{8}{9}$ is how many eighths?

If I cut $\frac{8}{9}$ of a yard of ribbon into 4 equal parts, how many ninths of a yard were in each part? If I had cut 8 yards of ribbon into 4 equal parts, how many yards would there be in each part? If $\frac{1}{4}$ of 8 yds. = 2 yds., then $\frac{1}{4}$ of $\frac{8}{9}$ will equal how many ninths?

$\frac{1}{8}$ of $\frac{8}{7}$ equals how many sevenths?

$\frac{1}{8}$ of $\frac{9}{11}$ equals how many elevenths?

$\frac{1}{8}$ of $\frac{13}{8}$ equals how many thirteenths?

$\frac{1}{4}$ of $\frac{8}{11}$ equals how many elevenths?

$\frac{1}{4}$ of $\frac{18}{8}$ equals how many eighteenths?

$\frac{1}{8}$ of $\frac{21}{8}$ equals how many twenty-firsts?

$\frac{1}{8}$ of $\frac{18}{8}$ equals how many twentieths?

$\frac{1}{7}$ of $\frac{21}{4}$ equals how many twenty-fourths?

§ 81. DIVISION OF FRACTIONS.

First Step. Divide fractions when the denominators are alike, and the numerator of the dividend is exactly divisible by the numerator of the divisor. Thus, $\frac{8}{9} \div \frac{2}{9}$; $\frac{9}{14} \div \frac{3}{14}$.

Second Step. Divide fractions when the denominators are alike, but the numerator of the dividend is not exactly divisible by the numerator of the divisor. Thus, $\frac{7}{8} \div \frac{3}{8}$.

Exercise for First Step.

I have here an orange cut into 12 equal parts. To how many children can you give $\frac{2}{12}$ of an orange, if you give $\frac{2}{12}$ to each child? In $\frac{9}{12}$ then how many times $\frac{2}{12}$?

John, you may take $\frac{8}{12}$ of the orange, and give $\frac{2}{12}$ to each child until you have none left. To how many children did you give $\frac{8}{12}$, giving $\frac{2}{12}$ to each child? How many times is $\frac{2}{12}$ found in $\frac{8}{12}$?

Annie, you may take $\frac{10}{12}$, and find how many times $\frac{5}{12}$ is found in $\frac{10}{12}$. Find how many times $\frac{2}{12}$ is found in $\frac{10}{12}$.

How many times is $\frac{2}{12}$ found in $\frac{6}{12}$? $\frac{4}{12}$ in $\frac{8}{12}$? $\frac{1}{12}$ in $\frac{9}{12}$? $\frac{6}{12}$ in $\frac{12}{12}$?

If I have $\frac{2}{3}$ of a dollar, and buy crackers at $\frac{2}{3}$ of a dollar a pound, how many pounds do I buy? If the crackers were $\frac{1}{3}$ of a dollar a pound, how many pounds could I buy?

If I have $\frac{2}{3}$ of a lb. of coffee, and I use $\frac{1}{3}$ each day, how many days will the coffee last? If I use only $\frac{1}{6}$ of a lb. a day, how many days will the coffee last? If I use $\frac{1}{6}$ a day, how many days will $\frac{2}{3}$ last?

If it takes me $\frac{2}{3}$ of an hour to write a line, how many times can I write the line in $\frac{2}{3}$ of an hour? If it takes me $\frac{2}{3}$ of an hour, how many times can I write the line? If it takes me $\frac{1}{3}$ of an hour to write a long line, how many times can I write it in $\frac{2}{3}$ of an hour? If James can copy his spelling in $\frac{2}{3}$ of an hour, how many times can he copy his spelling in $\frac{2}{3}$ of an hour? If it takes Nellie $\frac{2}{3}$ of an hour, how many times can she copy her words in $\frac{2}{3}$ of an hour? If it takes a car $\frac{1}{3}$ of an hour to make a trip, how many trips can the car make in $\frac{2}{3}$ of an hour?

Divide $\frac{2}{3}$ by $\frac{1}{3}$; $\frac{2}{3}$ by $\frac{2}{3}$; $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{2}{3}$.

Exercise for Second Step.

It takes $\frac{2}{3}$ of a yard of cloth for an apron. Who will tell me how many aprons I can make from $\frac{2}{3}$ of a yard? (3.) What part of a yard will remain after I take enough for 3 aprons? What part of enough for an apron is $\frac{1}{3}$ of a yard? How many aprons and what part of an apron can I make from $\frac{2}{3}$ of a yard, if it takes $\frac{2}{3}$ of a yard for each apron?

My bottle of ink is $\frac{2}{3}$ full. If I use $\frac{1}{3}$ of a bottle each month, how many months will the ink last me? (4 months.) At the end of four months, what part of a bottle of ink will remain? ($\frac{1}{3}$.) What part of a month will $\frac{1}{3}$ last me, if I use $\frac{1}{3}$ each month? How many months and what part of a month will $\frac{2}{3}$ of a bottle of ink last, if I use $\frac{1}{3}$ each month?

How much cake can I make from $\frac{1}{3}$ of a dozen eggs, if $\frac{1}{3}$ of a dozen are required for each pound of cake?

If a boy earns $\frac{3}{10}$ of a dollar a day, in what time will he earn $\frac{7}{10}$ of a dollar?

If a ship sails $\frac{5}{8}$ of a mile in a minute, in what time will it sail $\frac{3}{4}$ of a mile?

If a pound of coffee costs $\frac{6}{10}$ of a dollar, what amount of coffee can I buy for a dollar?

If crackers are $\frac{7}{10}$ of a dollar a pound, what amount of crackers can be bought for a dollar?

Divide $\frac{7}{10}$ by $\frac{3}{10}$; by $\frac{4}{10}$; by $\frac{5}{10}$;

Divide $\frac{8}{11}$ by $\frac{3}{11}$; by $\frac{4}{11}$; by $\frac{5}{11}$; by $\frac{6}{11}$.

Divide $\frac{15}{16}$ by $\frac{4}{16}$; by $\frac{5}{16}$; by $\frac{7}{16}$; by $\frac{9}{16}$.

Exercise for Review.

A slate pencil and a lead pencil together cost 4 cents. If the lead pencil cost three times as much as the slate pencil, what did each pencil cost?

A slate and a pencil together cost 9 cents. What did each cost, if the slate cost twice as much as the pencil?

A piece of baize for a desk cover and the braid to bind it, together cost 15 cents. If the braid cost only half as much as the baize, what did each cost?

I bought some pens and a penholder for 12 cents. The penholder cost twice as much as the pens. What did the pens cost?

I bought some stamps and an envelope for 10 cents. The stamps cost four times as much as the envelope. How much did each cost?

An apple and a peach together cost 6 cents. The peach cost twice as much as the apple. What did the apple cost?

Suppose you and I are 20 rods apart, and you gain upon me 2 rods every minute. How long before you will overtake me? If I can walk 20 rods in a minute, how far shall I walk while you are catching up to me? How far will you walk? How many more rods must I walk to go a mile?

If a bicycle goes 9 miles while I am walking 3 miles, how many times as fast as I does the bicycle go? While I am walking 2 miles, how far will the bicycle go? If the bicycle and I start together, and I go a mile and return, how many miles can the bicycle go and return in the same time?

If you are 9 years old and I am 21 years old, how old will each of us be when I am just twice as old as you?

If it costs 25 cents to send a telegram consisting of 10 words, and 2 cents for each additional word, what will it cost to send a message of 15 words?

If it costs \$5.00 for a table 6 feet long, and 50 cents for every additional foot, how much must I pay for a table $10\frac{1}{2}$ feet long?

If it takes 2 hours to roast a turkey weighing 10 pounds, and 15 minutes more for every additional pound, how long will it take to roast a turkey weighing 14 pounds?

How many tiles 4 inches square will it take to cover a space 16 inches square?

A meter is $39\frac{1}{8}$ inches. What is the difference in length between a yard and a meter? between 2 yards and 2 meters?

If you have $\frac{1}{2}$ of a dollar, and I have $\frac{1}{4}$ of a dollar, how many pounds of candy can we buy at $\frac{1}{4}$ of a dollar a pound?

If one man owns $\frac{1}{2}$ of a building lot and another man owns $\frac{3}{8}$ of the lot, how much of the building lot do both men own?

If I ride $\frac{1}{2}$ of a mile and walk $\frac{1}{8}$ of a mile, what part of a mile do I go?

If you study one lesson $\frac{1}{2}$ of an hour and another lesson $\frac{1}{10}$ of an hour, what part of an hour do you spend studying?

If there is $\frac{1}{2}$ of a dozen buttons on one card, $\frac{1}{4}$ of a dozen on another card, and $\frac{1}{8}$ of a dozen on yet another card, what part of a dozen is on all the cards together?

If I have $\frac{1}{2}$ of a card of matches and $\frac{1}{3}$ of a card, what part of a card have I all together?

If you have $\frac{1}{2}$ of a dollar and I have $\frac{1}{3}$ of a dollar, how much money have we together?

What is the sum of $\frac{1}{2}$ and $\frac{5}{6}$? $\frac{1}{3}$ and $\frac{5}{6}$? $\frac{2}{3}$ and $\frac{1}{3}$? $\frac{2}{3}$ and $\frac{3}{4}$? $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$? $\frac{2}{3}$ and $\frac{2}{3}$? $\frac{5}{6}$ and $\frac{2}{3}$?

If for dinner we eat $\frac{1}{2}$ and $\frac{1}{3}$ of the melon, what part of the melon is left?

I bought $\frac{5}{8}$ of a peck of sweet potatoes, and we have eaten $\frac{1}{2}$ of a peck. What part of a peck remains? How many more eighths of a peck ought I to have to make a whole peck?

If my bottle of ink is half full, and I use $\frac{2}{3}$ of a bottle of ink, what part of a full bottle will remain?

If from $\frac{3}{4}$ of a cask of vinegar $\frac{2}{3}$ of the cask is drawn off, what part of a full cask remains? If the cask holds 48 gallons, how many gallons remain in the cask?

If $\frac{1}{2}$ of a bushel of corn is taken from a basket holding $\frac{5}{8}$ of a bushel, what part of a bushel will remain?

What is the difference between $\frac{2}{3}$ and $\frac{2}{3}$? $\frac{1}{3}$ and $\frac{3}{4}$? $\frac{2}{3}$ and $\frac{1}{2}$? $\frac{5}{6}$ and $\frac{5}{6}$?

If you earn $\frac{2}{3}$ of a dollar a week, how much will you earn in 6 weeks? in 9 weeks? in 11 weeks?

How much will 6 books cost at $\frac{3}{4}$ of a dollar a book? at $\frac{2}{3}$ of a dollar a book? at $\frac{1}{2}$ of a dollar a book?

In one bunch of bananas there were 20 bananas. When $\frac{2}{3}$ of the bunch have been sold, how many bananas have been sold?

If a yard of cloth costs 48 cents, what must I pay for $\frac{3}{4}$ of a yard?

When apples are 60 cents a bushel, what must you pay for $\frac{7}{10}$ of a bushel?

In $\frac{5}{8}$ of a yard how many inches?

In $\frac{4}{5}$ of an hour how many minutes?

In $\frac{5}{8}$ of a bushel how many quarts?

A man cut $\frac{3}{8}$ of a cord of wood in a day, and his son cut $\frac{1}{2}$ as much. How many ninths did the son cut?

I have $\frac{2}{12}$ of a dozen eggs in one basket and $\frac{1}{3}$ as many eggs in another basket. What part of a dozen have I in the second basket?

In our back yard there are $\frac{3}{4}$ of an acre of land, and in the front yard there is only $\frac{1}{2}$ as much land. How many sevenths of an acre in the smaller yard?

I have a piece of ribbon $\frac{3}{4}$ of a yard long. If I cut off $\frac{1}{8}$ of the ribbon, what part of a yard shall I cut off?

I had in my purse $\frac{3}{12}$ of a dollar, but I spent $\frac{1}{4}$ of my money. What part of a dollar did I spend?

What is $\frac{1}{5}$ of $\frac{1}{11}$? $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{5}{15}$?

What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{2}{3}$?

When apples are $\frac{2}{3}$ of a dollar a peck, how many pecks can I buy for $\frac{4}{3}$ of a dollar?

If we use $\frac{2}{3}$ of a pound of butter at a meal, for how many meals will $\frac{3}{4}$ of a pound be sufficient?

If we use $\frac{3}{10}$ of a pound of coffee in a day, in how many days will we use $\frac{2}{10}$ of a pound?

If I eat $\frac{5}{11}$ of a pound of meat each meal, how many meals will $\frac{1}{11}$ of a pound last me?

If ice forms at the rate of $\frac{2}{8}$ of an inch an hour, how long will it take for the ice to be an inch thick? 6 inches thick? 9 inches thick? a foot thick?

If a vine grows $\frac{2}{10}$ of an inch a day, how many days will it take the vine to grow 9 inches?

A drover paid \$200 for lambs at \$5.00 apiece. How many lambs did he buy?

A man paid \$2.10 for eggs at \$0.30 a dozen. How many dozen of eggs did he buy?

I bought 3 bookcases, one for \$35.00, one for \$12.00, and one for \$8.00, and paid \$5.00 for having them var-

nished. At the end of 5 years I sold them for \$30.00. If they were worth \$6.00 a year to me while I had them, did I gain or lose?

I bought 40 yards of carpeting for \$0.75 a yard, and after using the carpet 5 years I sold it for \$15.00. If the carpet was worth \$3.00 a year to me, did I gain or lose?

A man bought a second-hand chamber set for \$25.00, and paid \$3.00 to put it in good order, then sold it for \$40.00. How much did he make?

If it takes \$84.00 to furnish a school-room with seats at \$2.00 each, how many are put into the room?

If the uniforms for a company of boys cost \$189.00, at \$3.00 each, how many boys are in the company?

If a merchant sells \$168.00 worth of boots in a week, at \$4.00 a pair, how many pairs does he sell? What does he gain at 25 cents a pair?

A man sold $\frac{4}{5}$ of a piece of cotton cloth which contained 45 yards. How many yards did he sell? How many yards were left?

A horse cost \$200, and a sleigh $\frac{2}{3}$ as much. What did the sleigh cost?

A man owned $\frac{3}{4}$ of a ship, and sold $\frac{1}{4}$ of his share. How many sixty-fourths had he left?

John had $\frac{4}{5}$ of a dollar, and George and Charles had each $\frac{7}{10}$ of a dollar. How much more had George and Charles together than John?

How many dollars will 3 barrels of flour cost at \$6 $\frac{2}{3}$ a barrel?

If 2 yards of cloth cost \$8 $\frac{1}{2}$, what is the price per yard?

How many pints in 12 $\frac{1}{2}$ gallons? How many gallons in 168 pints?

How many pecks in 248 quarts? How many bushels?

If a locomotive can go 8 miles in $\frac{1}{2}$ of an hour, how many miles can it go in one hour and a half?

CHAPTER XXXVII.

FACTORS.

§ 82. A FACTOR OF A NUMBER.

Name a number that will divide six without a remainder.
(2.)

Two is a *factor* of six.

Name another number that will divide six without a remainder. (3.)

Three is a factor of six.

Name another factor of six. (1.)

Name another factor of six. (6.)

Name a factor of *twelve*. (3.)

Name another factor of twelve. (4.)

Name another factor of twelve. (6.)

Name another factor of twelve. (2.)

Name the two remaining factors of twelve. (1 and 12.)

Name the factors of fifteen; of eight; of sixteen; of twenty; of twenty-four; of thirty-six; of forty-eight; of seventy-two.

What have you been naming of numbers?

What is true of a factor of a number?

§ 83. PRIME AND COMPOSITE NUMBERS.

What are the factors of seven? three? eleven? five? thirteen? seventeen? twenty-three? twenty-nine? thirty-one? forty-one?

Name another number of which the factors are simply one and the number itself; another; another.

Beginning with one, write in order a dozen numbers whose factors are simply one and the number itself.

Describe these numbers with reference to their factors.

All such numbers are **prime** numbers.

Describe a prime number. Name a number that has other factors as well as one and itself. Who thinks of three such numbers?

Write a score of such numbers.

Describe these numbers with reference to their factors.

All such numbers are **composite** numbers.

Describe a composite number. I will name the numbers from one to thirty, and you may describe each by saying prime or composite as I name.

Are most numbers prime or composite?

§ 84. PRIME FACTORS OF A NUMBER.

Name a prime number that is a factor of twelve. (2.)

Two is a **prime factor** of twelve.

Name another prime number that is a factor of twelve. (3.)

Three is a prime factor of twelve. Name a prime factor of sixteen; of eighteen.

Name two prime factors of twenty-one; of twenty-four; of twenty-eight; of thirty-six; of thirty-five.

What kind of factors have you been naming?

What do you mean by a prime factor of a number?

Write the prime numbers whose product is six; whose product is eight; whose product is twelve; whose product is sixteen; whose product is eighteen; whose product is twenty; whose product is twenty-four.

You have expressed the prime factors of these numbers.

Express the prime factors of thirty; of thirty-two; of thirty-six; of twenty-eight; of twenty-seven.

Complete this work:

The prime factors of 40 =

The prime factors of 42 =

The prime factors of 45 =

The prime factors of 44 =

The prime factors of 48 =

The prime factors of 49 =

The prime factors of 50 =

The prime factors of 54 =

The prime factors of 56 =

The prime factors of 60 =

The prime factors of 63 =

The prime factors of 64 =

The prime factors of 66 =

The prime factors of 70 =

The prime factors of 72 =

Find the product of the factors of each number, and see if the product is the number itself.

What are the prime factors of any number?

§ 85. A COMMON FACTOR OF TWO OR MORE NUMBERS.

Name a factor of four that is also a factor of six. (2.)

Two is a *common* factor of four and six.

Name a factor of four that is a factor of six, eight, and ten. (2.)

Two is a common factor of four, six, eight, and ten.

Name a factor of six that is a factor of nine and of twelve. (3.)

Three is a common factor of six, nine, and twelve.

Name a common factor of eight and twelve.

Name a common factor of six, nine, and eighteen.

Name a common factor of seven, fourteen, and twenty-one.

Name a common factor of ten, fifteen, and twenty-five.

Name a common factor of nine, twelve, fifteen, and twenty-four.

What is true of a common factor of two or more numbers?

Name all the factors common to six and twelve.

Which of these common factors is the greatest number?

Six is the **greatest common factor** of six and twelve.

Name all the factors common to twelve and twenty-four.

Which of these common factors is the greatest number?

Twelve is the **greatest common factor** of twelve and twenty-four.

Name all the factors common to eight, twelve, and twenty.

Which of these common factors is the greatest number?

What is four, then, of eight, twelve, and twenty?

Name the greatest common factor of six, twelve, and eighteen; of ten, twenty, and thirty; of eighteen and thirty-six; of fourteen and twenty-eight; of sixteen and twenty-four; of twenty-four, thirty, and thirty-six.

What is the greatest common factor of two or more numbers?

§ 86. A MULTIPLE OF A NUMBER.

Name a number of which two is a factor. (4.)

Four is a multiple of two.

Name another number of which two is a factor. (8.)

Eight is a multiple of two.

Name another multiple of two; another; another.

Beginning with four, write the multiples of two as far as thirty.

Write the multiples of three as far as thirty.

Write the multiples of four as far as forty.

Write **three** multiples of five ; of six ; of seven ; of eight ; of nine ; of ten.

What have you been writing ?

What do you mean by a multiple of a number ?

§ 87. A COMMON MULTIPLE OF GIVEN NUMBERS.

Name a multiple of two that is also a multiple of four.
(8.)

Eight is a **common multiple** of two and four.

Name another common multiple of two and four ; another ; another.

Name a common multiple of two, three, and four ; of two, four, and five ; of two, four, and six ; of three and five ; of three and seven ; of six and nine ; of two, three, four, six, eight, and twelve ; of two, three, four, six, nine, twelve, and eighteen.

What do you mean by a common multiple of two numbers ?

Name the smallest number that is a common multiple of two and three. (6.)

Six is the **least common multiple** of two and three.

Name the least common multiple of three and four ; of two and five ; of two and seven ; of three and five ; of two and four ; of three and six ; of four and eight ; of four and six ; of six and twelve ; of four and seven.

Name the least common multiple of two, four, and eight ; of three, six, and twelve ; of three, four, and six ; of two, four, and five ; of two, five, and ten ; of three, five, and ten ; of four, five, and ten ; of two, six, and ten ; of three, six, and ten.

Exercise for Review.

Express the factors of 12; of 18; of 24; of 36.

Point to the row of numbers which have 12 for a multiple.

Point to the row of numbers which have 24 for a multiple.

Point to the row of numbers which have 36 for a multiple.

Point to the row of numbers which have 18 for a multiple.

Draw a line through each number that is not found in *all* the rows.

What word will describe the factors that remain with reference to 12, 18, 24 and 36?

What word will describe 6 as a common factor of these numbers?

Try in the same way to find the **greatest common factor** of 14, 28, and 42.

Find the greatest common factor of 16, 24, 40, and 48.

Find the greatest common factor of 20, 40, 60, and 80.

Find the greatest common factor of 25, 50, and 75.

I have cloth out of which I wish to cut squares for a patchwork quilt. The cloth is of three widths: 12 inches, 18 inches, and 24 inches. What are the largest squares I can get without any waste of cloth?

A gardener has a rectangular flower-bed which is 6 feet on each of two sides, 8 feet on each of the other two sides, and 10 feet from corner to corner. If he borders this flower-bed with plants at equal distances apart, and sets them across the diagonals at the same distance apart, what is the greatest distance at which they can be placed from each other?

If the lengths of the sides and diagonal were 4, 3, and 5

respectively, what would be the greatest distance at which plants could be placed to be the same distance apart?

If the lengths were 12, 16, and 20, what would be the greatest distance?

If the lengths were 24, 32, and 40, what would be the greatest distance?

I have some paper which is 9 inches wide, some which is 12 inches wide, and some which is 15 inches wide, which I wish to cut into strips of equal width. What can be the greatest width of the strips so as to have no waste of the paper?

Write the numbers in order from 1 to 30.

Draw a line through each *odd* number.

Place a dot over each *prime* number.

Is every odd number a prime number?

What even number is a prime number?

What word will describe all numbers not prime?

Are all composite numbers even numbers?

Are all even numbers composite numbers?

Are there more composite numbers than even numbers?

Name the prime factors of 6, 8, 9, 12, 18, 20, 24.

Divide 48 by one of its prime factors. Divide the answer by a prime factor. Go on dividing by prime factors until the answer is a prime number. Find the product of the last answer and the several divisors. What is the answer?

What then are the prime factors of 48?

In the same way find the prime factors of:

72	148	196	289
144	200	225	324
96	169	256	400

Write in a row the multiples of 2 as far as 30.

Write the multiples of 3 as far as 30.

Write the multiples of 4 as far as 28.

Draw a line through every multiple not found in all the rows.

What word will describe the remaining multiples with reference to 2, 3, and 4?

What word will describe 12 as a common multiple of 2, 3, and 4?

Write the multiples of 6, 9, and 4 until you find the **least common multiple** of the numbers.

Find the least common multiple of 2, 3, 4, and 8.

Find the least common multiple of 3, 5, and 2.

Find the least common multiple of 4, 5, and 8.

I have here several disks of paper. You will see that one is divided into halves, another into thirds, another into fourths, and so on, each being divided into a certain number of equal parts.

I wish you to show me $\frac{1}{2}$ and $\frac{1}{4}$ of the same disk. Which disk will you choose? (The one divided into fourths.)

Which disk will you choose to show me $\frac{1}{2}$ and $\frac{1}{3}$ of the same one? (The one divided into sixths.)

Which disk will you choose to show me $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

Which disk to show me $\frac{1}{2}$ and $\frac{1}{3}$?

Which disk to show me $\frac{1}{3}$ and $\frac{1}{4}$?

Which disk to show me $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$?

Which disk to show me $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{12}$?

Which disk to show me $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$?

Which disk to show me $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{12}$?

Which disk to show me $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$?

I have blocks which are 3, 4, and 6 inches long. What must be the length of the shortest box into which they exactly fit?

Find the shortest length of plank that I must buy if I wish to cut it into pieces 3 feet, 6 feet, or 9 feet long, so that there will be no waste.

Find the narrowest width of cloth that I must buy if I wish to cut it into 2 inch squares, 3 inch squares, 4 inch squares, 6 inch squares, 8 inch squares, or 12 inch squares, without any waste.

CHAPTER XXXVIII.

DECIMALS.

§ 88. TENTHS.

Name a fraction whose denominator is ten. Express the fraction on the board. ($\frac{3}{10}$.)

I will express the same fraction. (0.3.)

Read what I have expressed.

Name another fraction whose denominator is ten. Express. ($\frac{7}{10}$.)

I will express the same fraction. (0.7.)

Express one-tenth in this new way. (0.1.)

We call it the *decimal* way.

Express nine-tenths the decimal way. (0.9.)

Express two-tenths, four-tenths, five-tenths, six-tenths, eight-tenths.

Read what you have expressed as I point.

Express and add :

One-tenth, three-tenths, four-tenths.

Five-tenths, two-tenths, one-tenth.

Seven-tenths, one-tenth, zero-tenths.

Eight-tenths, zero-tenths, one-tenth.

Six-tenths, two-tenths, one-tenth.

Express and subtract :

Eight-tenths minus three-tenths.

Nine-tenths minus seven-tenths.

Seven-tenths minus three-tenths.

Nine-tenths minus six-tenths.

Nine-tenths minus five-tenths.

Express and multiply :

Four-tenths by two.

Two-tenths by three.

Six-tenths by one.

Three-tenths by two.

Two-tenths by four.

One-tenth by seven.

Express and divide :

Eight-tenths by two-tenths.

Six-tenths by three-tenths.

Eight-tenths by four-tenths.

Nine-tenths by three-tenths.

Six-tenths by two-tenths.

Express and find :

One-half of four-tenths; of eight-tenths; of six-tenths;
of two-tenths.

One-third of six-tenths; of three-tenths; of nine-tenths.

One-fourth of eight-tenths; of four-tenths.

Read what I have expressed. ($32\frac{3}{10}$.)

Who will express the same number and write the fraction
the decimal way. (32.3.)

Write decimally :

Seven and six-tenths.

Six and five-tenths.

Eighteen and nine-tenths.

Twelve and seven-tenths.

Twenty and eight-tenths.

Fifty and one-tenth.

Three hundred and twenty-four and four-tenths.

Six hundred and thirty and two-tenths.

Express and add :

Eight-tenths, seven-tenths, five-tenths, four-tenths, seven
tenths, three-tenths.

Six-tenths, nine-tenths, eight-tenths, five-tenths, seven-tenths, four-tenths.

Three-tenths, seven-tenths, nine-tenths, three-tenths, six-tenths, two-tenths.

Eleven and three-tenths, five and seven-tenths, eighteen and nine-tenths, twenty and eight-tenths.

Express and subtract :

Seven and five-tenths minus seven-tenths.

Fifteen and three-tenths minus eight-tenths.

Twenty-five and two-tenths minus sixteen and nine-tenths.

Forty-two and five-tenths minus twenty and six-tenths.

Sixty-five and seven-tenths minus thirty and eight-tenths.

Express and multiply :

Eight-tenths by seven.

Nine-tenths by eight.

Six-tenths by eight.

Seven-tenths by nine.

Eight and seven-tenths by six ; by eight.

Twelve and five-tenths by fourteen ; by nineteen.

Express and divide :

One and eight-tenths by nine-tenths.

Twenty-seven and three-tenths by three-tenths.

Fifty-six and eight-tenths by eight-tenths.

Eighty-five and six-tenths by four-tenths.

Seventy-seven and five-tenths by five-tenths.

Ninety-eight and four-tenths by six-tenths.

§ 89. TENTHS AND HUNDREDTHS.

Name a fraction whose denominator is hundredths.

Express it on the board. ($\frac{17}{100}$.)

I will express the same fraction. (0.17.)

Read what I have expressed.

Express in this decimal way :

Eighteen-hundredths; twenty-four hundredths; fifty-two hundredths; seventy-four hundredths; sixty-nine hundredths; ninety-seven hundredths.

Read what you have expressed as I point.

Look at this number. (0.13.) When I hide the three so that you see only the point and one, what will you call the number you see? (0.1.) One-tenth equals how many hundredths? Thirteen-hundredths are how many more than one-tenth? Read the number then in tenths and hundredths.

Read in tenths and hundredths each of the number of hundredths you have just expressed on the board.

Read this number in tenths and hundredths. (0.04.) This number. (0.05.)

Express :

Seven-hundredths; eight-hundredths; one-hundredth; three-hundredths; two hundredths; six-hundredths; nine-hundredths.

Read what you have expressed.

Express and add :

Three-hundredths, eight-hundredths, nine-hundredths, five-hundredths, seven-hundredths.

Nine-hundredths, six-hundredths, five-hundredths, seven-hundredths, five-hundredths.

Sixteen-hundredths, seven-hundredths, thirteen-hundredths, thirty-six hundredths, four-hundredths, fifteen-hundredths.

Twenty-three hundredths, nineteen hundredths, seventeen-hundredths, eight-hundredths, forty-hundredths, fifty-hundredths.

Three and five-hundredths, seven and six-hundredths, nine and fifteen-hundredths, six and fifty-four hundredths, seven and sixty-three hundredths.

Express and subtract :

Nine-hundredths minus three-hundredths.

Seven-hundredths minus four-hundredths.

Fifteen-hundredths minus seven-hundredths.

Seventeen-hundredths minus nine-hundredths.

Forty-two hundredths minus twenty-eight hundredths.

Ninety-five hundredths minus sixty-seven hundredths.

Eight and seventy-two hundredths minus five and fifty-four hundredths.

Seven and eighty-three hundredths minus four and thirty-eight hundredths.

Seventy-two and ninety-four hundredths minus thirty-five and twenty-six hundredths.

Express and multiply :

Nine-hundredths by seven.

Eight-hundredths by nine.

Seven-hundredths by eight.

Twelve-hundredths by seven.

Fifty-seven hundredths by nine.

Seventy-eight hundredths by eight.

Nine and sixty-seven hundredths by forty-six.

Six and forty-nine hundredths by seventy-seven.

Express and divide :

Nine-hundredths by three-hundredths.

Twenty-seven hundredths by nine-hundredths.

Forty-two hundredths by seven-hundredths.

Fifty-six hundredths by eight-hundredths.

Eighty-seven hundredths by three-hundredths.

Ninety-six hundredths by sixteen-hundredths.

One and thirty-six hundredths by seventeen-hundredths.

One and seventy-six hundredths by sixteen-hundredths.

Express and add :

Seventy-five and eight-tenths, five and six-hundredths, three-tenths, eight and sixty-seven hundredths, nine-hundredths, seven and eight-tenths.

Fifty-nine and sixty-eight hundredths, seventy-seven and nine-hundredths, eighty-five and seven-tenths, ninety and six-hundredths, six and ninety-seven hundredths, eight-hundredths, nine-tenths, five, eighty.

Express and subtract :

Twenty minus seven-tenths.

Fifty minus eighteen-hundredths.

Sixty minus nine-hundredths.

Seventy and two-tenths minus nineteen-hundredths.

Seven-tenths minus sixty-seven hundredths.

Express and multiply :

Eight by seven-tenths.

Sixteen by nine-tenths.

Sixty-nine by eight-tenths.

Seven by nine-hundredths.

Eighty-seven by sixty-nine hundredths.

Five-tenths by seven-tenths.

One and seven-tenths by nine-tenths.

Two and eight-tenths by three and seven-tenths.

Seven and nine-tenths by eight and seven-tenths.

Express and divide :

Two by one-tenth.

Four by two-tenths.

Eight by four-tenths.

Twenty-seven by three-tenths.

Two-tenths by one-hundredth.

Six-tenths by three-hundredths.

Seven and two-tenths by eight-hundredths.

Thirty-seven and five-tenths by fifteen-hundredths.

Nine by eighteen-hundredths.

Seven by fourteen-hundredths.

Eight by sixteen-hundredths.

Five and one-tenth by seventeen-hundredths.

Seven and two-tenths by twenty-four hundredths.

Express either in tenths or in hundredths:

One-half.

One-twentieth.

One-fifth.

Three-twentieths.

Two-fifths.

Seven-twentieths.

Three-fifths.

Nine-twentieths.

Four-fifths.

One twenty-fifth.

One-fourth.

Twelve twenty-fifths.

Three-fourths.

One-fiftieth.

Find :

0.3 of 25.

0.06 of 76.

0.7 of 63.

0.09 of 89.

0.9 of 72.

0.03 of 80.

0.5 of 84.

0.05 of 100.

0.4 of 95.

0.06 of 100.

0.2 of 79.

0.06 of 200.

0.6 of 96.

0.12 of 450.

0.1 of 800.

0.16 of 325.

0.6 of 125.

0.25 of 484.

0.8 of 376.

$0.33\frac{1}{3}$ of 100.

0.9 of 284.

$0.66\frac{2}{3}$ of 100.

0.7 of 576.

$0.16\frac{2}{3}$ of 100.

Express as common fractions, and reduce to lowest terms :

0.5	0.2	0.8	0.08	0.04	0.12	0.16	0.50
0.4	0.6	0.02	0.06	0.05	0.15	0.25	0.75

Express :

Two dollars and thirty-six cents (\$2.36); twelve cents (\$0.12); one dollar and five cents; five dollars and one cent; three dollars and twenty-one cents; ten dollars and fifty cents; fifty dollars and ten cents.

Read what you have expressed.

Express and add :

Three dollars and twenty-five cents, one dollar and fifty cents, nine cents, thirty-six cents, five dollars and thirty cents.

Express and subtract :

Six dollars and fifty cents minus three dollars and thirty-two cents.

Eight dollars and eighty-two cents minus four dollars and forty-eight cents.

Five dollars minus two dollars and sixty-two cents.

Express and multiply :

Seven dollars and fifteen cents by seven.

Eleven dollars and four cents by four.

Eight dollars and ten cents by eight.

Express and divide :

Ten dollars and twelve cents by two.

Five dollars and twenty-two cents by three.

Ten dollars and eighty-four cents by five dollars and forty-two cents.

Three hundred and twenty-four dollars and eight cents by one dollar and seven cents.

Six hundred and fifty dollars and nine cents by one dollar and two cents.

Nine hundred dollars by eighty dollars.

CHAPTER XXXIX.

PERCENTAGE.

§ 90. PERCENTAGE.

Express on the board one-hundredth; four-hundredths; ten-hundredths; twenty-hundredths; twenty-five hundredths; fifty-hundredths; seventy-five hundredths.

What have you been expressing?

I have a new name for a *hundredth*. I will write it. (*per cent.*) What is it?

One per cent is how many hundredths?

Two per cent is how many hundredths?

Eight per cent is how many hundredths?

Fifty per cent is how many hundredths?

One-hundredth is what per cent?

Three-hundredths is what per cent?

Fifteen-hundredths is what per cent?

Read what you have written, giving the name *per cent*.

Write what you have read, using the words *per cent*.

Read these per cents, giving the name *hundredths*:

25 per cent.	60 per cent.	80 per cent.
40 per cent.	90 per cent.	2 per cent.
100 per cent.	200 per cent.	500 per cent.
$33\frac{1}{3}$ per cent.	$66\frac{2}{3}$ per cent.	$12\frac{1}{2}$ per cent.

Two per cent of fifty dollars is how many hundredths of fifty dollars?

Two per cent of fifty dollars is how many dollars?

Twenty-five per cent of eighty dollars is how many hundredths of eighty dollars?

Twenty-five per cent of eighty dollars is how many dollars?

Find five per cent of sixty; ten per cent of ninety; seventy-five per cent of twenty-four; sixty per cent of twenty; fifty per cent of one hundred and fifty.

If from a barrel holding 40 gallons of vinegar 15 per cent is drawn off, how many gallons are drawn off?

A salesman has 250 pieces of cloth. If 4 per cent of the number of pieces is imperfect, how many pieces are imperfect?

If of a flock of 300 sheep 3 per cent died, how many sheep died?

I have in my school building 350 children. If on a stormy day $14\frac{2}{3}$ per cent of the number is absent, how many are absent?

If of 50 words which you write 2 per cent is wrong, how many words are misspelled?

If I give you 25 examples to perform, and you have time for only 80 per cent of that number, how many do you perform?

I have a box containing 12 bunches of envelopes. If I use $16\frac{2}{3}$ per cent of the envelopes, how many bunches do I use?

If a man buys a house for \$20,000, and sells it for 100 per cent of its cost, for what does he sell it?

If you have 20 words to spell, and spell 100 per cent of them correctly, how many do you spell correctly?

If my watch cost \$50, and I sell it for 100 per cent of its cost, what do I gain?

If thread which costs 5 cents a spool is sold for 100 per cent of its cost, what is the loss per cent?

If a man sells a horse for 100 per cent of its cost, what does he gain?

What is 100 per cent of any number?

Any number is what per cent of itself?

If the price of a book is \$1.50, but the salesman deducts 2 per cent of the price, how much does he get for the book?

If I lend a man \$200, and he pays me 6 per cent of it for the use of it, how much money must he pay me when he settles the note?

If you go to school 75 per cent of the year, how many months do you have for vacation?

For what must cloth which cost a dollar a yard be sold to gain 1 per cent?

For what must apples which cost 12 cents a dozen be sold to gain 50 per cent?

For what must confectionery which costs 20 cents a pound be sold to gain 100 per cent?

For what must soap which costs 32 cents a cake be sold to gain 25 per cent?

A basket of wood which weighed 50 pounds stood out in the rain. When wet it was found to have gained 20 per cent in weight. How much did it weigh then?

A quantity of sugar which weighed 75 pounds was found to have decreased 40 per cent in weight by drying. How much did it then weigh?

Express in all the ways you can:

1 per cent.	5 per cent.	$16\frac{2}{3}$ per cent.
2 per cent.	$12\frac{1}{2}$ per cent.	20 per cent.
4 per cent.	$14\frac{2}{3}$ per cent.	25 per cent.

Here is a sign which we use instead of the words *per cent* (%).

Express each of the above per cents on the board, using this sign.

In how many ways can you now express a per cent?

What kind of a fraction is a per cent?

Express each of these per cents in the form of a common fraction reduced to lowest terms :

25%.	$16\frac{2}{3}\%$.	100%.	8%.
20%.	75 %.	10%.	150%.
50%.	$12\frac{1}{2}\%$.	200%.	5%.

Express each of these numbers as a per cent :

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$, 1, 2, 3, 8, $1\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{1}{8}$, $1\frac{1}{10}$, $1\frac{1}{20}$, $1\frac{1}{25}$, $1\frac{1}{50}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{2}{5}$, $\frac{3}{7}$, $1\frac{3}{4}$, $1\frac{2}{3}$, 1.

If you have 4 apples, and eat 2, what part of the number of apples have you eaten? What per cent have you eaten?

2 is what per cent of 4?

If you have 20 cents, and spend 10 cents, what part of your money do you spend? What per cent of your money?

10 is what per cent of 20?

A merchant has a piece of flannel containing 25 yards. If he sells $12\frac{1}{2}$ yards, what fractional part of the whole does he sell? What per cent of the whole does he sell?

$12\frac{1}{2}$ is what per cent of 25?

If it is 50 miles between two places, and you start from one of the places, and travel 25 miles toward the other, what part of the distance do you travel? What per cent of the distance do you travel?

25 is what per cent of 50?

If I teach 6 hours in the day, what part of the day do I teach? What per cent of the day am I employed in teaching?

6 is what per cent of 24?

If I use 18 sheets from a quire of paper, what part of the whole do I use? What per cent of the whole?

18 is what per cent of 24?

If I have four 5-cent pieces, and spend one of them, what part of my money do I spend? If I spend 3 pieces, what per cent do I spend?

5 is what per cent of 20?

15 is what per cent of 20?

If you have ten 2-cent pieces, and spend 2 of them, what part of your money do you spend? What per cent of your money do you spend? If you spend 5 of them, what per cent of your money do you spend?

4 is what per cent of 20?

10 is what per cent of 20?

If a man earns \$25 a week, and spends \$25 a week, what per cent of his earnings does he spend?

25 is what per cent of 25?

If a man sells a load of wood for \$15, which cost him \$15, what per cent of the cost is the selling price?

15 is what per cent of 15?

If you buy a knife for \$1, and sell it for \$1, what per cent of the cost is the selling price?

1 is what per cent of 1?

40 is what per cent of 40?

50 is what per cent of 50?

90 is what per cent of 90?

4 is what per cent of 4?

2 is what per cent of 2?

100 is what per cent of 100?

200 is what per cent of 200?

If a man buys shoes for \$5 a pair, for what must he sell them to receive a hundred per cent of their cost?

5 is 100% of what number?

If a man buys sugar at 8 cents a pound, for what must he sell it per pound to receive 100% of its cost?

8 is 100% of what number?

12 is 100% of what number?

25 is 100% of what number?

100 is 100% of what number?

A jeweller bought a pin for \$2, which was 50% of what he received for it. For what did he sell it?

2 is 50% of what number?

Ida spent 6 cents this morning, which was 50% of all the money she had. How much money had she?

6 is 50% of what number?

9 is 50% of what number?

50 is 50% of what number?

100 is 50% of what number?

200 is 50% of what number?

In your class there are 12 children, which is 25% of the whole number in the room. How many are in the room?

12 is 25% of what number?

A man gained 20 cents on a book, which was 25% of the cost of the book. What was the cost of the book?

20 is 25% of what number?

7 is 25% of what number?

9 is 25% of what number?

25 is 25% of what number?

50 is 25% of what number?

100 is 25% of what number?

In selling ladies' hose, a merchant lost 1 cent on a pair, which was 10% of the cost. What was the cost?

On a summer's day a pint of water evaporated from a tank. If this was 5% of the water in the tank, how much water was there in the tank?

At the top of our blackboard there is a strip 8 inches in width which is painted green. If this strip is 20% of the whole width of the blackboard, how wide is the blackboard?

If a man pays \$3 a month for milk, and this is 4% of the table expenses for the month, what are the table expenses for the month?

A man paid \$2 for the use of some money for one month, and this was 1% of the sum he borrowed. What sum did he borrow?

2 is 4% of what number?

5 is 5% of what number?

1 is 2% of what number?

1 is 1% of what number?

8 is 10% of what number?

7 is 20% of what number?

I bought a book-case for \$15, and sold it for \$18. What per cent did I gain?

There were 20 children in the class at the beginning of the term, but there are only 15 now. What per cent of the class has dropped out?

A man who buys apples at 2 cents sells them at 3 cents. What per cent does he gain?

If bananas cost 4 cents, and sell at 7 cents, what is the per cent of gain?

If envelopes cost 6 cents a bunch, and sell for 12 cents a bunch, what is the per cent of gain?

If cloth which costs \$1 a yard sells for \$1.05, what is the per cent of gain?

If goods which cost \$100 sell for \$75, what is the per cent of loss?

If a man puts \$1000 into business at the beginning of the year, but finds at the end of the year that he has but \$800, what is his per cent of loss?

If 75% of gun-powder is saltpetre, what per cent of the whole is the rest of the mixture?

If 25% of a barrel of oil is drawn off, what per cent is left in the barrel?

If a man loses 10% in weight, what per cent of his former weight is his present weight?

If I spend 90% of my income for the year, what per cent of my income do I save?

If only 50% of grape wine is pure juice of the grape, what per cent is adulteration?

If coffee is adulterated 15%, what per cent is pure coffee?

If to reduce the strength of vinegar I put water with it so that 85% only is vinegar, what per cent is water?

If 70% of the rock in New England is granite, what per cent is not granite?

If in a mixture of green and black tea 30% is green tea, what per cent is black tea?

If a man sells goods at 120% of the cost, what per cent does he gain?

If goods are marked down to 75% of their cost, what per cent of the cost is the reduction?

§ 91. COMMISSION AND BROKERAGE.

I wish to hire you to sell some fruit for me, and this is the bargain I will make with you:

You shall receive 2% of the amount of money you bring me from the sale of the fruit.

If you sell 20 crates of peaches, at \$2.50 a crate, what will you receive for your wages?

If you sell 1000 boxes of grapes, at 20 cents a box, what shall I pay you?

How much will you receive for selling:

100 barrels of apples, at \$2 a barrel?

5 dozen apricots, at 15 cents a dozen?

10 dozen oranges, at 3 cents an orange?

40 boxes of raspberries, at 25 cents a box?

80 boxes of strawberries, at 15 cents a box?

20 dozen bananas, at 2 cents each?

Do you know of any one who works on this plan?

When any person is engaged to work for another on these terms, we call him a *commission agent*.

The money which he receives is called his *commission*.

You sold fruit for me on a commission.

I will make you my agent to sell other goods on a commission.

If you sell \$1500 worth of butter, at 4% commission, what shall I pay you? What shall I receive for the butter?

If you sell \$500 worth of boots and shoes, at a commission of 3%, what shall I receive for the goods?

If you sell \$350 worth of books, at a commission of $2\frac{1}{2}\%$, and pay \$5 for freight on the books, what shall I receive from the sale of the books?

What ought you to receive for buying \$900 worth of goods for me, at a commission of $1\frac{1}{2}\%$?

If an agent buys \$2550 worth of fruit, at a commission of $3\frac{1}{2}\%$, what does he receive for his work?

If an agent sells 25 sewing machines, at \$40 each, and receives a commission of $1\frac{3}{4}\%$ besides his expenses, which are \$50, how much does the owner receive from the sale of the machines?

An agent received \$3000 to invest in flour, at \$5 a barrel. How many barrels could he buy after deducting his commission of 2%? For what must the merchant sell the flour per barrel so as to receive 100% of the cost? For what must he sell the flour so as to gain 20% of its cost?

If a man sells a house for me for \$1000, what do I receive of the amount if his commission is 4%? If, besides my agent's commission, I pay \$25 for advertising, what has it cost me to sell my place?

An agent sold 5000 pounds of cotton, at 10 cents a pound. After deducting his commission of $2\frac{1}{2}\%$, what amount of money had he which belonged to his employer? He bought cotton cloth with this money, and received $2\frac{1}{2}\%$ commission for buying. After deducting his commission for buying, how many yards of cloth, at 6 cents a yard, could he buy with the sum that remained?

§ 92. INTEREST.

A man offers to take you as a partner in his business if you will contribute \$1000 to the capital already in the business. You have but \$500, so you decide to borrow the other \$500. Can you have the use of that money for nothing? What do you think you must do about it? You are right: you must pay for the use of it. I will say you pay \$30 for the use of it. Who knows what name we give to the sum you pay for the use of the money? What *interest* do you pay on the \$500?

What do you mean when you say you pay \$30 *interest*?

When you borrow this money, the man of whom you borrow does not say, "I will lend it to you for \$30"; he says, "I will lend it to you at 6% interest." What is the interest on \$500, at 6%?

Suppose you have to pay 7% interest on \$500, to what will the interest come?

To what will it come if you have to pay 5% interest? 9% interest? 4% interest? 3% interest? 8% interest?

If at 6% you pay only \$30 for the use of \$500, do you know how long you have a right to keep the money?

You are right: you can keep it but 1 year. Rate of interest is usually reckoned by the year.

What must you pay if you keep the \$500 2 years? 3 years? 4 years? 5 years? $\frac{1}{2}$ of a year? $\frac{1}{3}$ of a year? $\frac{2}{3}$ of a year? $\frac{1}{4}$ of a year? $\frac{1}{5}$ of a year? 6 months? 4 months? 2 months?

The amount of interest which you must pay on \$500 depends upon two conditions. Can you name the two conditions?

At 6%, what is the interest on \$250 for 2 years? for $\frac{1}{2}$ of a year? for 3 months? for 3 years and 6 months? At 7%, what is the interest for the time mentioned in each case? Find the interest at 5%.

Find the interest of:

1. \$150 for 2 years 6 months, at 6%.
 2. \$310.60 for 3 years 4 months, at 6%.
 3. \$60.25 for 4 years 2 months, at 7%.
 4. \$100 for 6 years 6 months, at 8%.
 5. \$420 for 6 months, at 5%.
 6. \$550.50 for 1 year 3 months, at 8%.
 7. \$836.75 for 2 years 4 months, at 6%.
 8. \$740 for 3 years 2 months, at $4\frac{1}{2}\%$.
 9. \$860.40 for 6 years 4 months, at 6%.
 10. \$999.95 for 1 year 9 months, at 6%.
 11. \$275 for 4 months, at $7\frac{1}{2}\%$.
 12. \$350 for 4 years 8 months, at 6%.
 13. \$200.50 from Nov. 19 to March 19, at 6%.
 14. \$600 from July 3 to Jan. 3, at 8%.
-

If you keep \$500 one year, at 6% interest, what amount of money must you pay when you settle the note? If you keep the money 2 years, and make no payments of interest during the time, what amount must you pay when you settle the note?

To what will \$300 amount in 4 years, at 6% interest?

To what will \$120.25 amount in $3\frac{1}{2}$ years, at 7% interest?

To what will \$2000 amount in 5 years, at 4% interest?

How do you find the amount when you know the interest?

What do you mean by *amount*?

The sum of money which is borrowed has a particular name. Can you tell me what it is? It is called the *principal*.

Name the principal in the examples I have just given you to perform.

Which requires less time, to find the interest or to find the amount of any principal?

Which is the greater sum?

Find the amount of:

1. \$50 for 3 years 2 months, at 6%.
2. \$35 for 4 years 6 months, at 7%.
3. \$90 for 1 year 3 months, at 6%.
4. \$125 for 2 years 8 months, at 6%.
5. \$375 for 6 years 9 months, at $4\frac{1}{2}\%$.
6. \$480 for 5 years 3 months, at 8%.
7. \$720 for 7 years, at 4%.
8. \$650 for 8 years 4 months, at 6%.
9. \$300 for 2 years 2 months, at 6%.
10. \$525.50 for 1 year 9 months, at 6%.
11. \$250 from Jan. 1, 1880, to July 1, 1886, at 6%.
12. \$125.25 from March 3, 1882, to Oct. 3, 1884, at 6%.
13. \$75.75 from May 10 to Sept. 10, at 6%.
14. \$85 from Oct. 24 to April 24, at 6%.
15. \$90.25 from Dec. 20 to Sept. 20, at 7%.
16. \$70.75 from May 1 to Jan. 1, at 8%.

I paid \$24 for the use of \$200 for 2 years. I want you to help me find the rate of interest. If I paid \$24 for 2 years, what was the interest for 1 year? (\$12.) \$12 is what part of \$200? ($\frac{3}{50}$.) $\frac{3}{50}$ equals what per cent? (6%.) What then was the rate of interest?

Could you have found what per cent 12 was of 200 without reducing the fraction?

I paid \$63 interest on \$350 for 3 years. Can you find the rate of interest? What was the interest for 1 year? (\$21.) 21 is what per cent of 350? (6%.) What rate of interest did I pay?

If the interest on \$200 for 6 years is \$84, what is the rate per cent? For what time will you find the interest on the principal first? What will you do next? What is your answer? (7%.)

The interest on \$500 for 6 months is \$20. I wish to know the rate per cent. What is the interest on the principal for 1 year? (\$40.) 40 is what per cent of 500? (8%.) What is the rate of interest?

Find the rate per cent :

1. When the interest on \$50 for 2 years is \$6.
2. When the interest on \$405.25 for 1 year is \$16.21.
3. When the interest on \$600 for 4 years is \$120.
4. When the interest on \$425.25 for 6 months is \$17.01.
5. When the interest on \$375 for 4 months is \$7.50.
6. When the interest on \$25 for 1 year is \$2.
7. When the interest on \$75.50 for 10 years is \$30.20.
8. When the interest on \$100 for 1 year is \$3.
9. When the interest on \$200 for 6 months is \$3.
10. When the interest on \$50 for 2 years is \$3.
11. When the interest on \$450 for 3 years is \$60.75.
12. When the interest on \$600 for 3 months is \$9.
13. When the interest on \$1000 for 1 year is \$60.
14. When the interest on \$850 for 2 years is \$102.
15. When the interest on \$725.50 for 9 months is \$43.53.
16. When the interest on \$300 for 8 months is \$12.

I borrowed \$600, at 5% interest. If the interest amounted to \$60 for the time I kept the principal, can you tell me how long I kept it? You may first tell me what would be the interest on \$600, at the given rate, for 1 year. (\$30.) If the interest for 1 year is \$30, how many years will it require to make the interest \$60? (2 years.) How long then did I keep the principal?

If the interest on \$25, at 8%, was \$4, how long was the principal on interest? What will you first find? Having found the interest for 1 year to be \$2, what will you do to find how many years the principal was on interest to gain \$4?

The interest on \$425.25, at 8%, was \$51.03. How long was the money on interest? What was the interest for 1 year? (\$34.02.) How long then was the money on interest to gain \$51.03? ($1\frac{1}{2}$ years.)

A man paid \$60.75 for the use of \$450, at $4\frac{1}{4}\%$ interest. How long did he keep the principal? For what length of time will you first find the interest? What then will you do to find the time?

Find the time in which :

1. The interest on \$200 will amount to \$30, at 6%.
2. The interest on \$275.50 will amount to \$38.57, at 6%.
3. The interest on \$475 will amount to \$99.75, at 7%.
4. The interest on \$500 will amount to \$82.50, at $4\frac{1}{2}\%$.
5. The interest on \$60 will amount to \$168.55, at $7\frac{1}{2}\%$.
6. The interest on \$100 will amount to \$22, at 6%.
7. The interest on \$50 will amount to \$40, at 8%.
8. The interest on \$1000 will amount to \$975, at $7\frac{1}{2}\%$.
9. The interest on \$850 will amount to \$136, at 6%.
10. The interest on \$700 will amount to \$70, at 5%.

11. The interest on \$625.25 will amount to \$75.03, at 4%.
 12. The interest on \$575.50 will amount to \$17.26½, at 6%.
 13. The interest on \$350.50 will amount to \$12.22, at 8%.
 14. The interest on \$75.50 will amount to \$20.74, at 7%.
 15. The interest on \$90 will amount to \$4.50, at 7½%.
 16. The interest on \$900 will amount to \$54, at 6%.
-

I kept a sum of money 2 years, and paid \$6 interest at 6%. I want you to tell me what the sum was. You may first tell me what I would pay for \$1 for the given time at the given per cent. (12 cents.) If \$1 gained 12 cents, how many dollars must I have had to gain \$6 interest? (\$50.)

A certain sum of money produced \$99.75 in 3 years, at 7%. Let us see if we can find the sum. What would \$1 produce in the given time at the given rate? (21 cents.) If \$1 produced 21 cents, what sum was required to produce \$99.75? (\$475.)

What sum of money must I put in the bank that in 10 years will produce \$600, at 6%? What will you first find? Having found that \$1 for the given time at the given rate produces 60 cents, how will you find what sum will produce \$600 in the same time at the same rate? What sum do you find? (\$1000.)

Find the principal that :

1. Will produce \$24 interest in 3 years, at 4%.
2. Will produce \$45 interest in 2½ years, at 6%.
3. Will produce \$17.50 interest in ½ year, at 7%.
4. Will produce \$84 interest in 2½ years, at 6%.

5. Will produce \$128 interest in 4 years, at 8%.
 6. Will produce \$31.50 interest in 9 months, at 6%.
 7. Will produce \$22.50 interest in 3 years, at $7\frac{1}{2}\%$.
 8. Will produce \$35 interest in $1\frac{1}{4}$ years, at 8%.
 9. Will produce \$75 interest in 2 years, at 5%.
 10. Will produce \$231 interest in 7 years, at 4%.
 11. Will produce \$297 interest in $5\frac{1}{2}$ years, at 6%.
 12. Will produce \$13.75 interest in 1 year, at $5\frac{1}{2}\%$.
 13. Will produce \$8 interest in $\frac{3}{4}$ year, at 8%.
 14. Will produce \$51.25 interest in $3\frac{1}{4}$ years, at 6%.
-

Just a year ago to-day I put a sum of money in the bank, at 6% interest. That sum now amounts to \$106. I want you to find what sum I put in the bank. If I had put in \$1, what would it have amounted to? (\$1.06.) If \$1 would have amounted to \$1.06, can you not tell what sum I must have deposited to amount to \$106? (\$100.)

I have just deposited a sum of money in the bank, at 6% interest. If it remains there $2\frac{1}{2}$ years, it will amount to \$230. What is the sum of money which I have just deposited? (\$200.)

If to-day you put out, at 7% interest, a sum which in 3 years will amount to \$363, what is the sum at interest? (\$300.)

What sum in 3 years and 4 months, at 9% interest, will amount to \$325? (\$250.)

When John was 5 years old, his father put a sum of money in the bank for him, at 6% interest. When he was 12 years old, the sum had amounted to \$213. What was the sum deposited? (\$150.)

If, at the age of a year and four months, a sum of money was put at interest for me at 6%, and when I was 18 years old it had amounted to \$1000, what was the sum? (\$500.)

Henry has a sum of money given him on his fifth birthday, which he lent to his father, at 6% interest. In 16 years 8 months it had amounted to \$10. What was the sum given him? (\$5.)

If to-day I invest a sum of money in railroad bonds, at 7% interest, which will amount to \$745 in 7 years, can you tell me the sum I invest? (\$500.)

A man has a sum of money invested in mortgages which yield him 1% a month. If in 8 years 4 months the sum amounts to \$2000, what is the sum invested? (\$1000.)

If you have a sum of money invested in government bonds which yields you 5% interest, and in 5 years it amounts to \$1000, what is the sum invested? (\$800.)

Find the principal that will amount :

1. To \$868 in 4 years, at 6%.
2. To \$1083 in 3 years 6 months, at 4%.
3. To \$555 in 5 years 4 months, at 9%.
4. To \$795 in 9 months, at 8%.
5. To \$918 in 2 months, at 12%.
6. To \$404 in 3 months, at 4%.
7. To \$598 in 8 months, at 6%.
8. To \$4000 in 12 years 6 months, at 8%.
9. To \$2000 in 16 years 8 months, at 6%.

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